Wavelets, Multi-Resolution and Fast Z Pinch Research

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Outline

- Wavelets & Multi-Resolution Analysis
- Bolometry Data Denoising: Energy Dips and Power
- Turbulent Mix Data Analyzed Using Wavelets: How Important Are Initial Perturbations
- Mach II 2D MRT Instability Break up Analyzed Using Wavelets: Compression Far Beyond that of FFTs

What are Wavelets Commonly Used For and Why?

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- **Signal processing**: Flexible, efficient signal representation and decomposition: Beat FT, WFT, CT, DCT, etc. The High Road in DSP (Stephane Mallat)
- **Data Compression**: FBI fingerprint archives 26:1 wavelet based compression, otherwise, at 500 pixels/inch and 256 levels of gray-scale information per pixel, one crook = 6MBytes, entire FBI database = 200 Terabytes (30 Mega-suspects) @ $1000/Gbyte = $ 200 Megabucks! Sparse representations: Average data (smooth, well represented) + details (successively ignored) => Subband Coding
- **Denoising**: Recovering Brahms himself playing Hungarian dance number 1 in 1889. Hear it @ http://www.music.yale.edu Shrinkage and Thresholding: Keep sharp features, lose the noise. Ask the expert Donoho@stanford.edu
- **Pattern Detection**, self similarity, coherent structures: See El Nino's regularity for yourself in Chi ^2 distribution of wavelet power => time series had Gaussian statistics: http://paos.colorado.edu/research/wavelets/wavelet1.html

How Do We Use Wavelets and MRA in Z Pinch Research?

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- Adaptive Grids, rezoning and remeshing fast O(N) algorithms
- Tracking the plasma in phase space as opposed to gridding phase space uniformly (finally get to >1D and up to 3x + 3v Vlasov)
- Compressed representations of multiscale MRT data (experimental or numerical) "optimum representations" which automatically denoise the signal as well. Pinhole and Backlighting data analysis.
- Tracking turbulence across scales. Criteria for degree of turbulence and mix by which to compare different Z shots.
- Denoising bolometry data and extracting power from noisy X ray energy data
- Any intermittent, spiky, nonsteady behavior is ill served by Fourier analysis (misses the point) but well served by wavelets.

What Are Wavelets?
Start @ (www.wavelets.org) & Surf (Mathsoft, amara, ...)

Mallat, Meyer, Daubechies, Beylkin, Coifman, Strang, Sweldens, Donoho...

- Wavelets are localized kernels or atoms in PHASE SPACE.
- You may think of them as basis functions with prescribed dilation and translation properties.
- They may or may not be orthonormal or have compact support or be differentiable everywhere, or be fractal, or have many zero moments.
- Wavelets are like breathing wave packets which can home in on structures in phase space better than FT or WFT ever could.

$$\psi_{j,k}(x) = 2^{j/2}\,\psi\left[2^j\left(x - \frac{k}{2^j}\right)\right]\quad j,k \in Z$$

$$\Psi_n(x) = (-1)^n \frac{d^n}{dx^n}\left[\exp\left(-\kappa\,(x - x_c)^2/2\right)\right]$$

When the scale is decreased translation steps between wavelets should likewise be decreased

What is MRD or Multi-resolution Decomposition?

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- Multiresolution: Zoom in and out on a number of successively finer scales in a sequence of nested approximation subspaces $\{V_j\}_{j \text{ in } Z}$.
- In general, get an overcomplete basis set in $L_2(R)$. Approximate (or truncate) by bounding the scales of interest.

Scaling functions and the scaling equation:

Low pass filter

$$\varphi(x) = 2\sum_{k=0}^{2N-1} h_k \varphi(2x-k)$$

$$\sum_k h_k = 1 \qquad \int_{-\infty}^{\infty} \varphi(x)\,\mathrm{d}x = 1$$

The Wavelets:

High pass filter

$$\psi(x) = 2\sum_{k=0}^{2N-1} g_k \varphi(2x-k)$$

$$g_k = (-1)^k h_{2N-1-k}$$

These filters decompose a sampled signal into 2 sub-sampled channels: the coarse approximation of the signal and the missing details at finer scales. The original signal can be reconstructed from these channels by interpolation.

Discrete Wavelet Transforms Perfect Reconstruction Subband Coding Filters

DWTs are Orthonrmal decompositions:

$$f(t) = \sum_{k} c_k \phi_k(t) + \sum_{j=0}^{J} \sum_{k} d_{jk} \psi_{jk}(t)$$

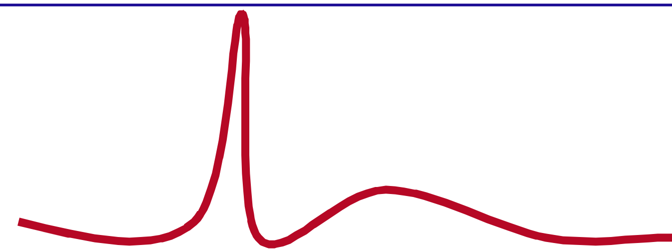
$$c_m = \int f(t)\,\phi_m(t)\,dt, \quad d_{lm} = \int f(t)\,\psi_{lm}(t)\,dt$$

The number of operations required to perform DWTs with a filter of length L (with L taps) is of order L x N (Even FFTs require N ln N operations)

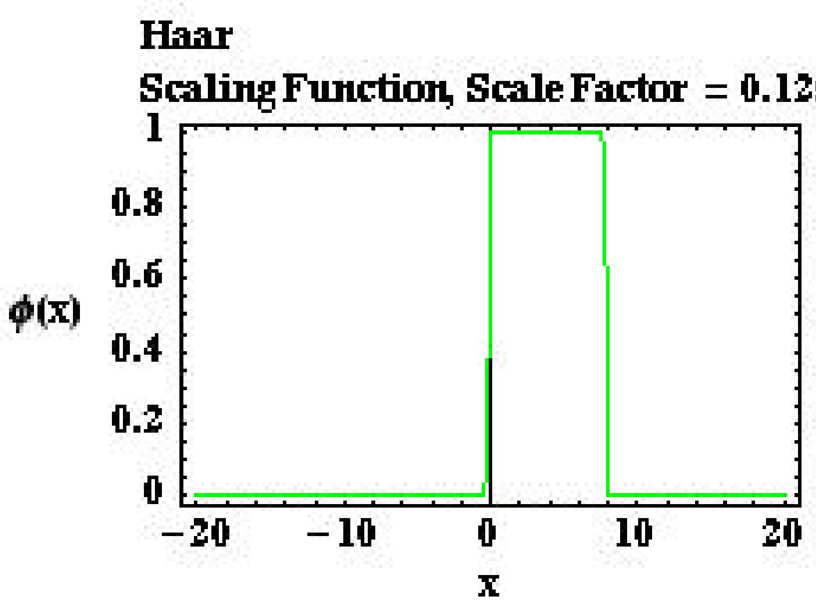
$$L\,N\left(1 + \frac{1}{2} + \frac{1}{2^2} + \ldots\right) < 2LN$$

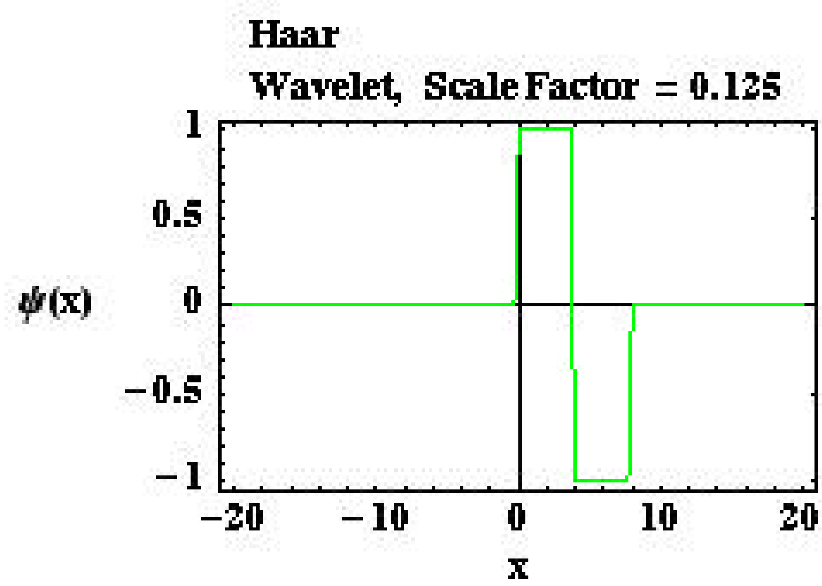
The Key to Multi-Rresolution Analysis Using Wavelets Is:

- **THRESHOLDING**
- **Two Ways to do it:**
- **Linear or Scale Thresholding**
- **Nonlinear or Largest Coefficient Thresholding**
- **Linear is Fourier like: Keep up to some scale and chop off the rest**
- **Nonlinear Thresholding is the true breakthrough: Keep those wavelets which have the largest coefficients no matter where they are and on whatever scale they are. No need to keep intermediate scales or intermediate locations. Just keep the BIG stuff. Automatically denoise, automatically compress and automatically bring out significant patterns.**



The Scaling Function and Wavelet for Haar or Daubechies 1 in X-Space





The Scaling Function and Wavelet for Haar or Daubechies 1 in K- Space

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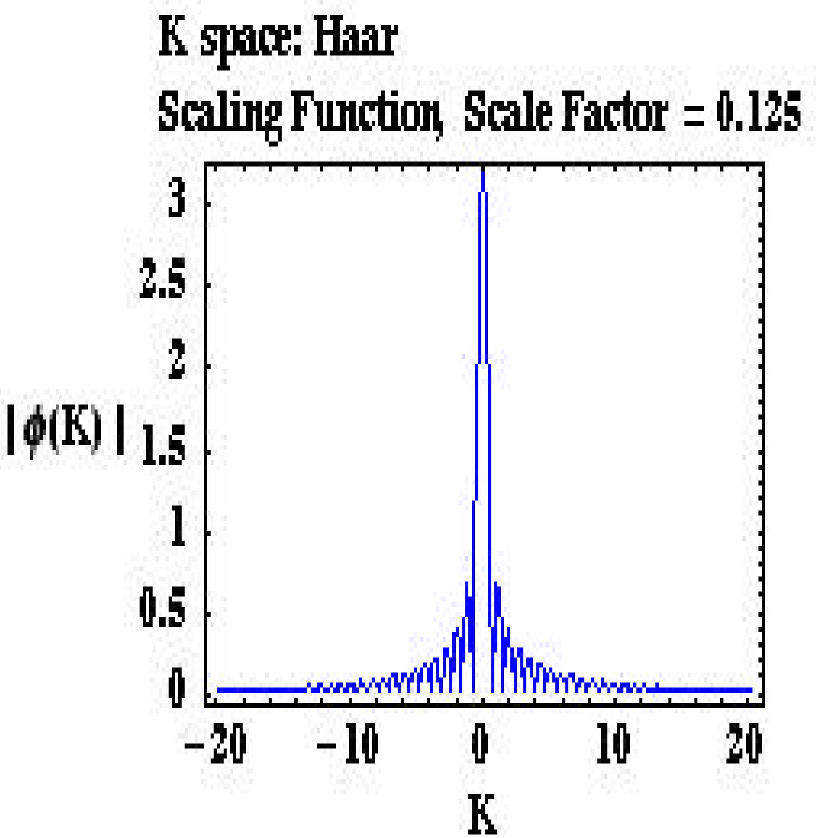

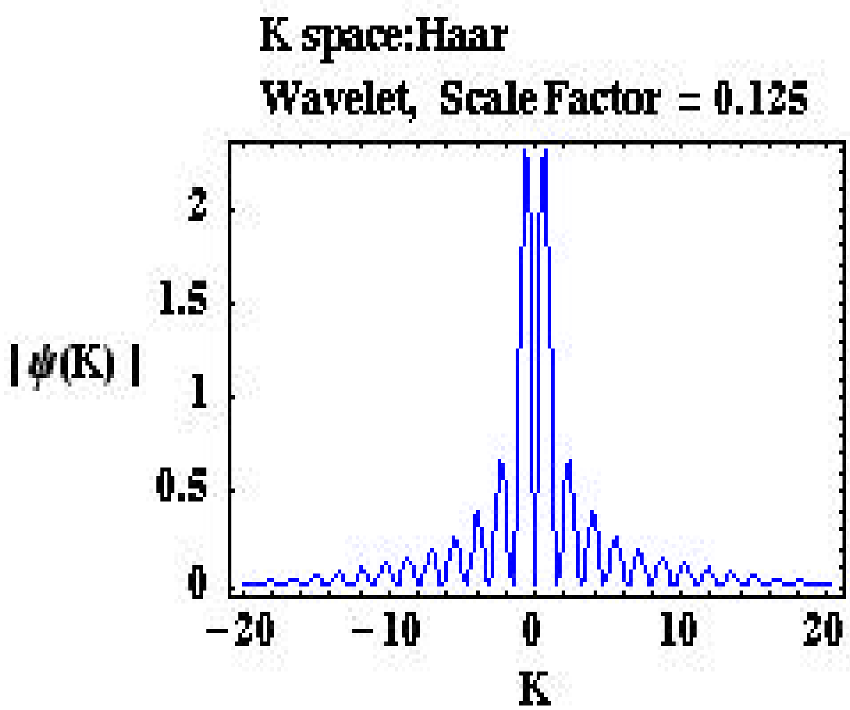

The Scaling Functions and Wavelets for Daubechies 2-6 in X-Space

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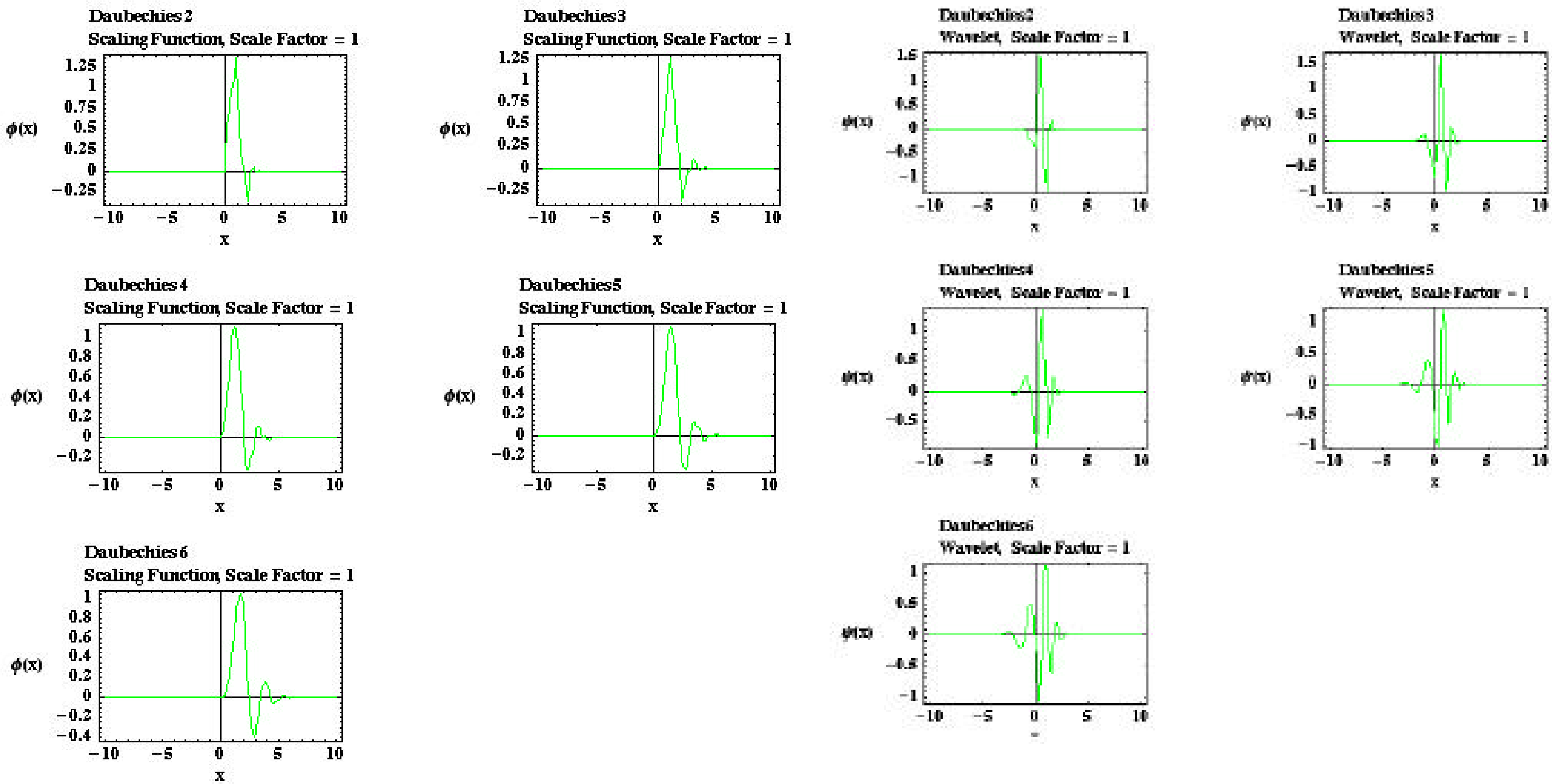

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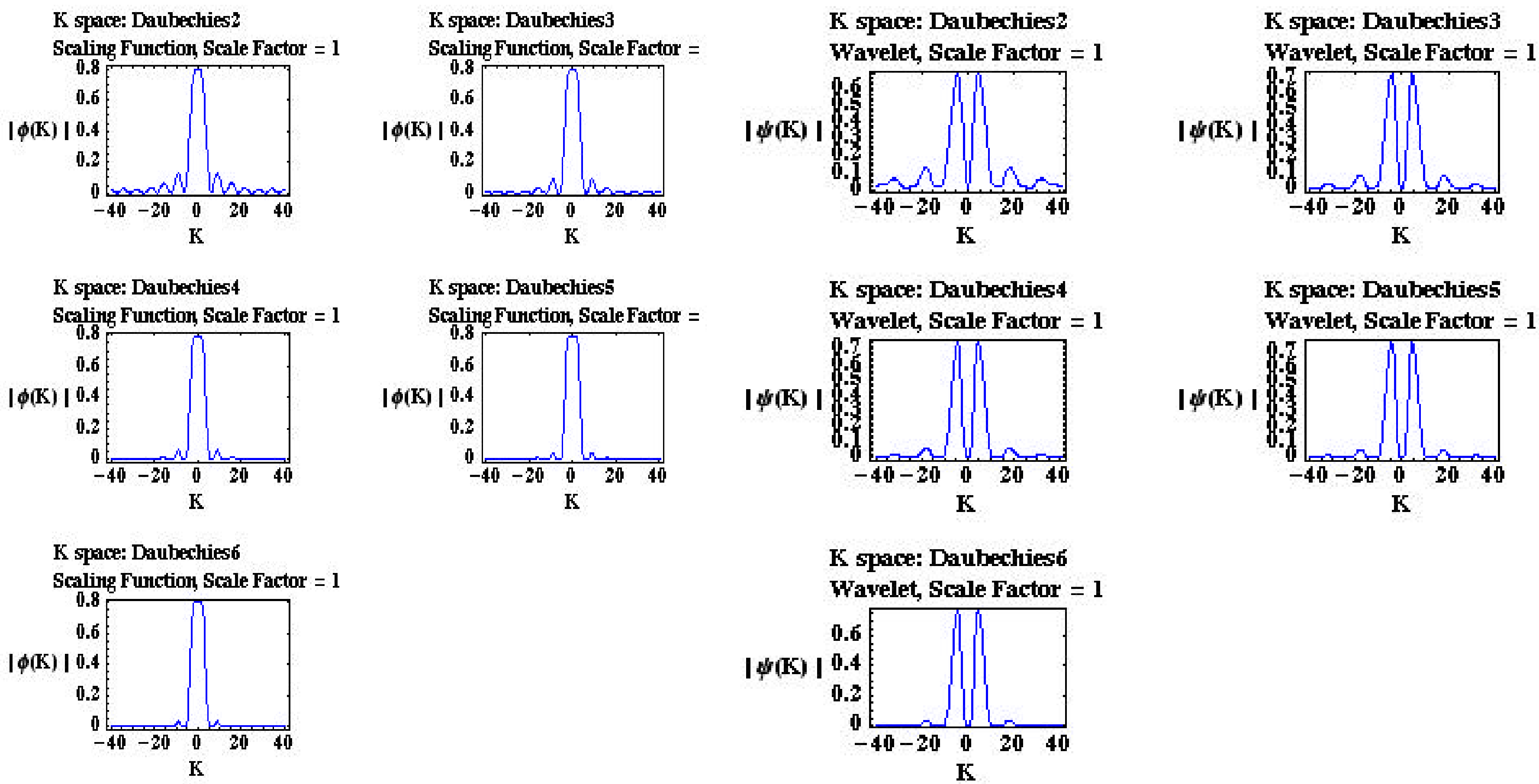

Z66 Bolometer signal: Cropped version (800 points) + Doubled for Periodicity (1600 points) + FFT of resulting signal

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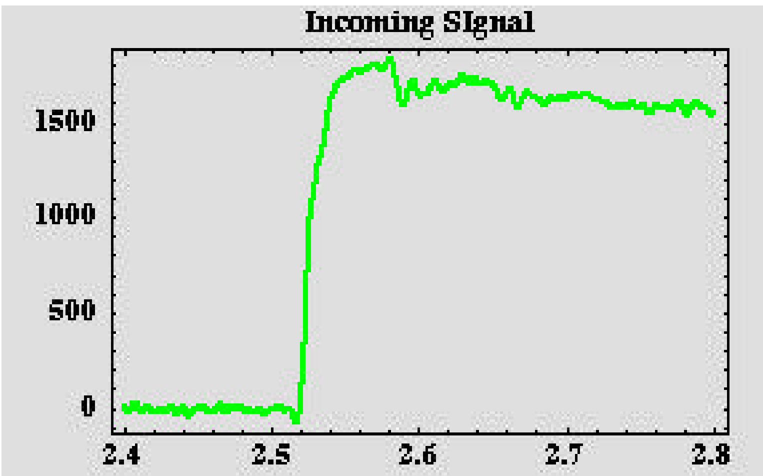

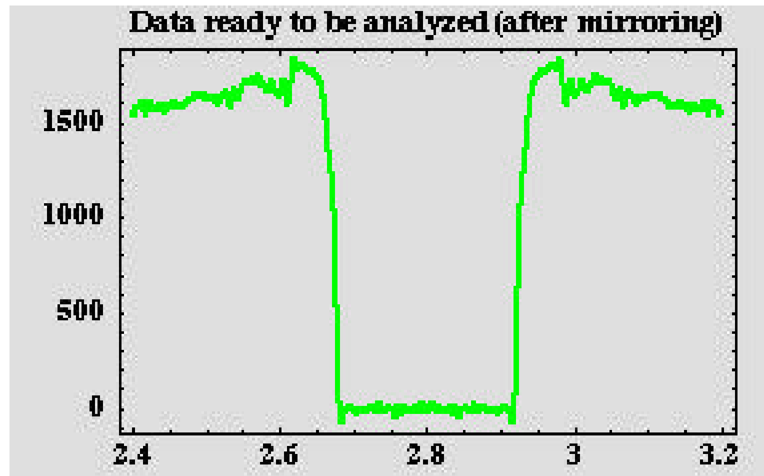

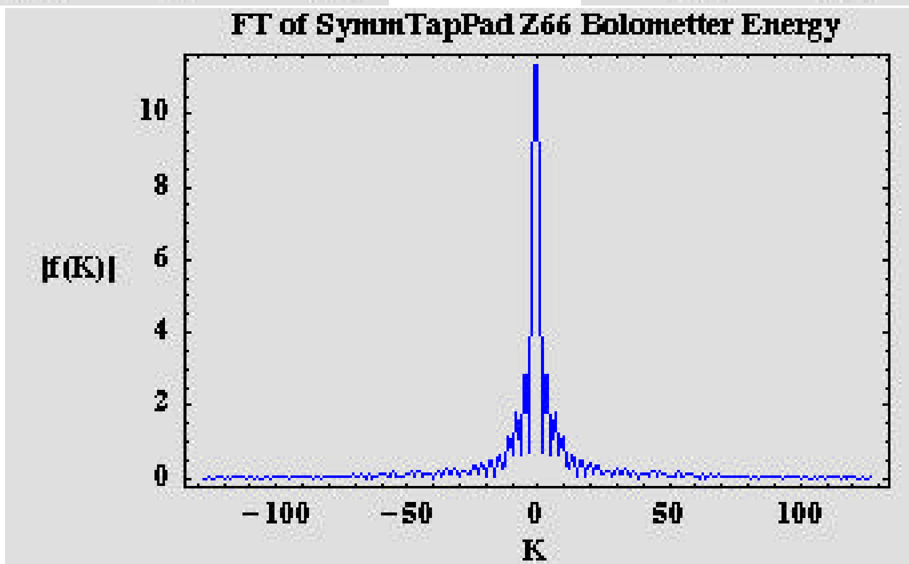

MRD Coefficients of Z66 Bolometry Signal with 4 different filters

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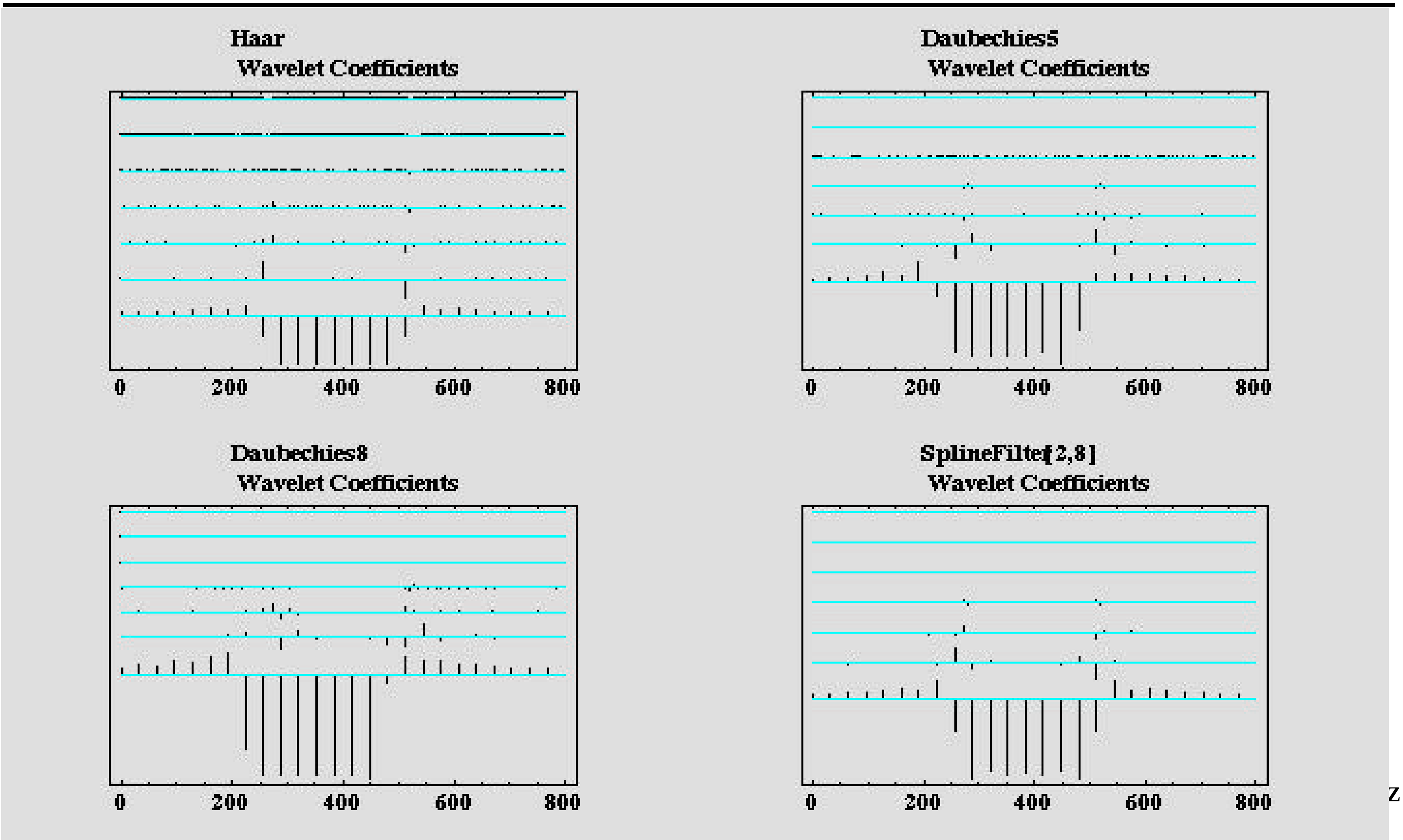

The Actual MRD at Different Levels with 4 Different Filters

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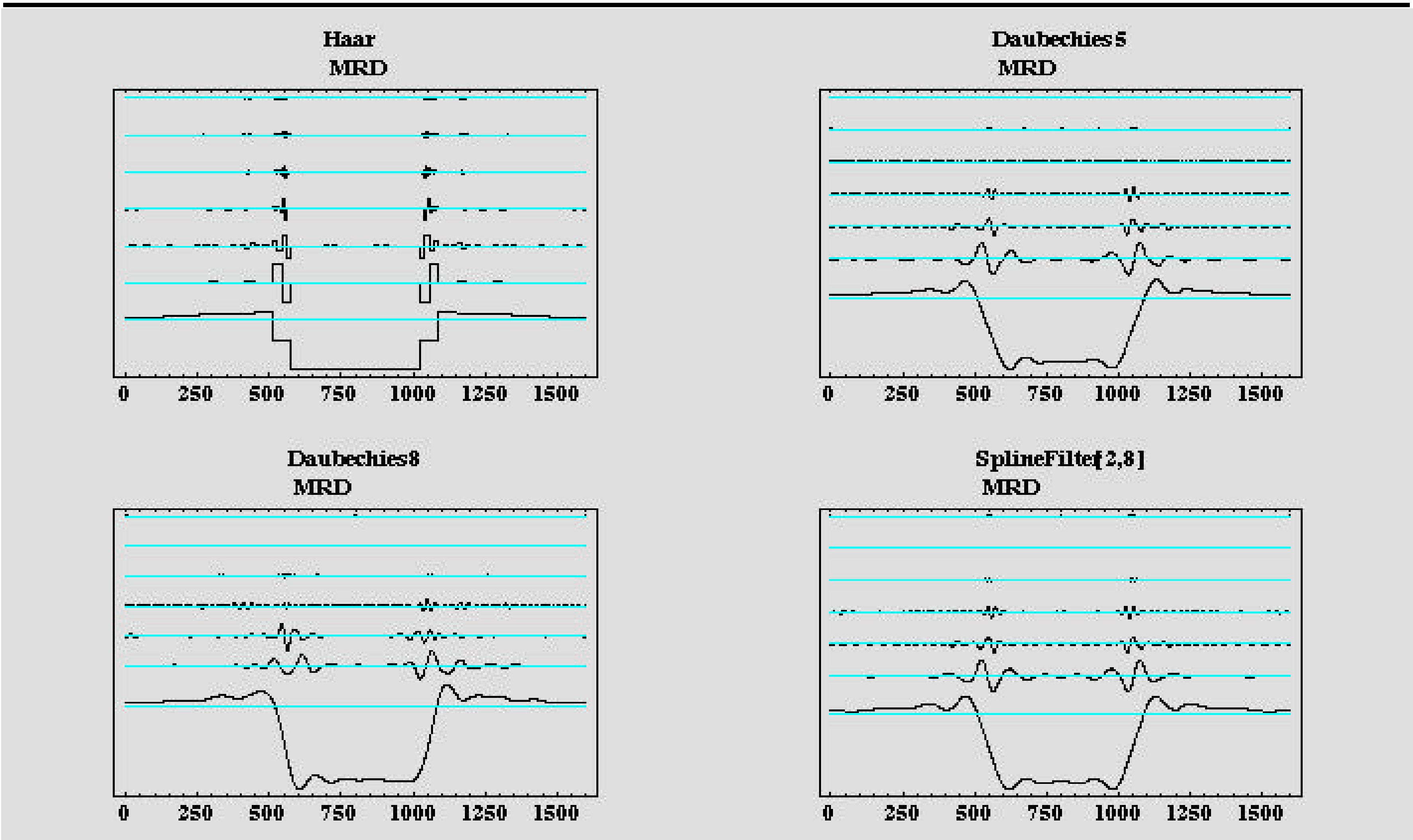

The Rate of Decay of the Largest Coefficients of the MRD

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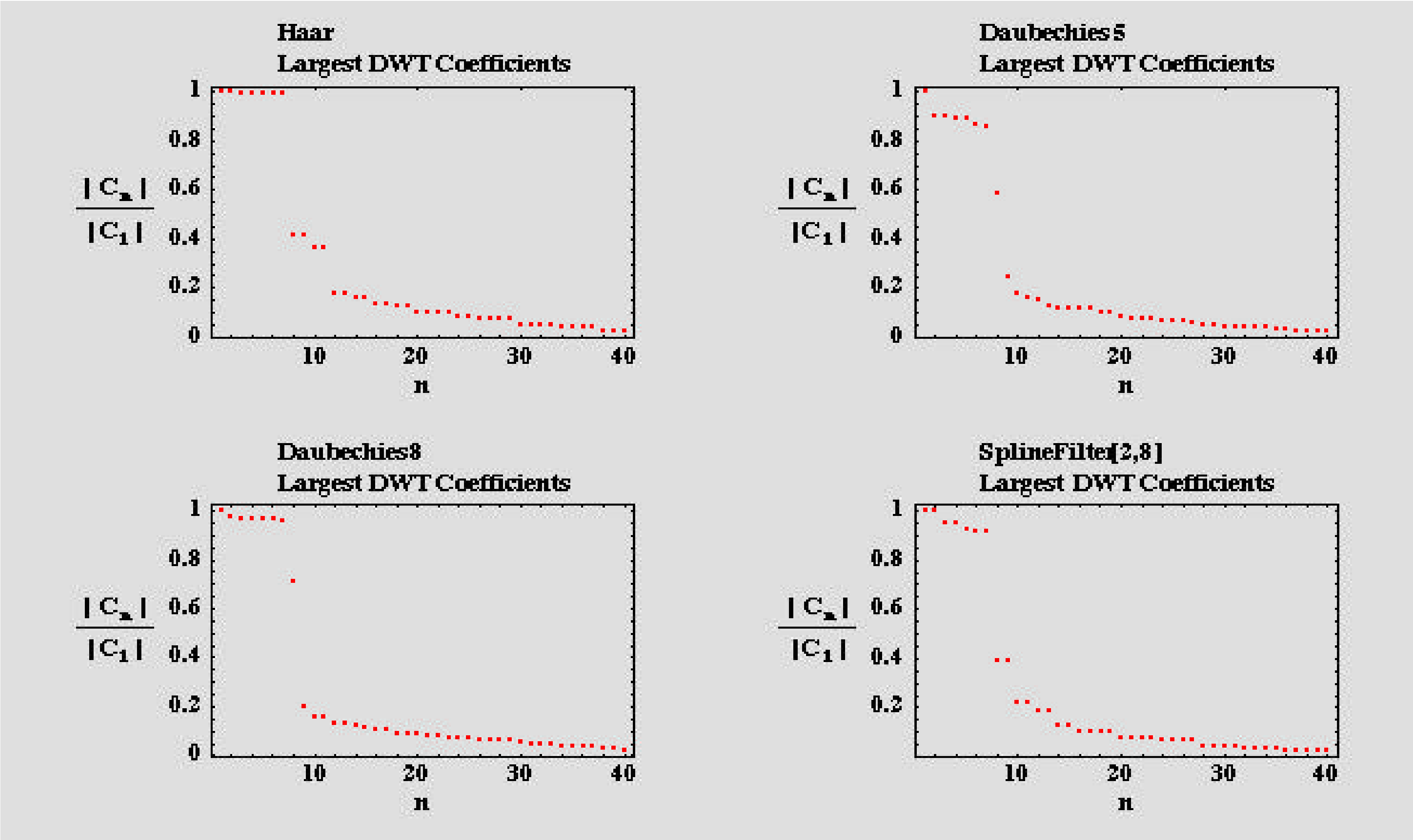

The Energy Accumulation Rate in the Largest Coefficients

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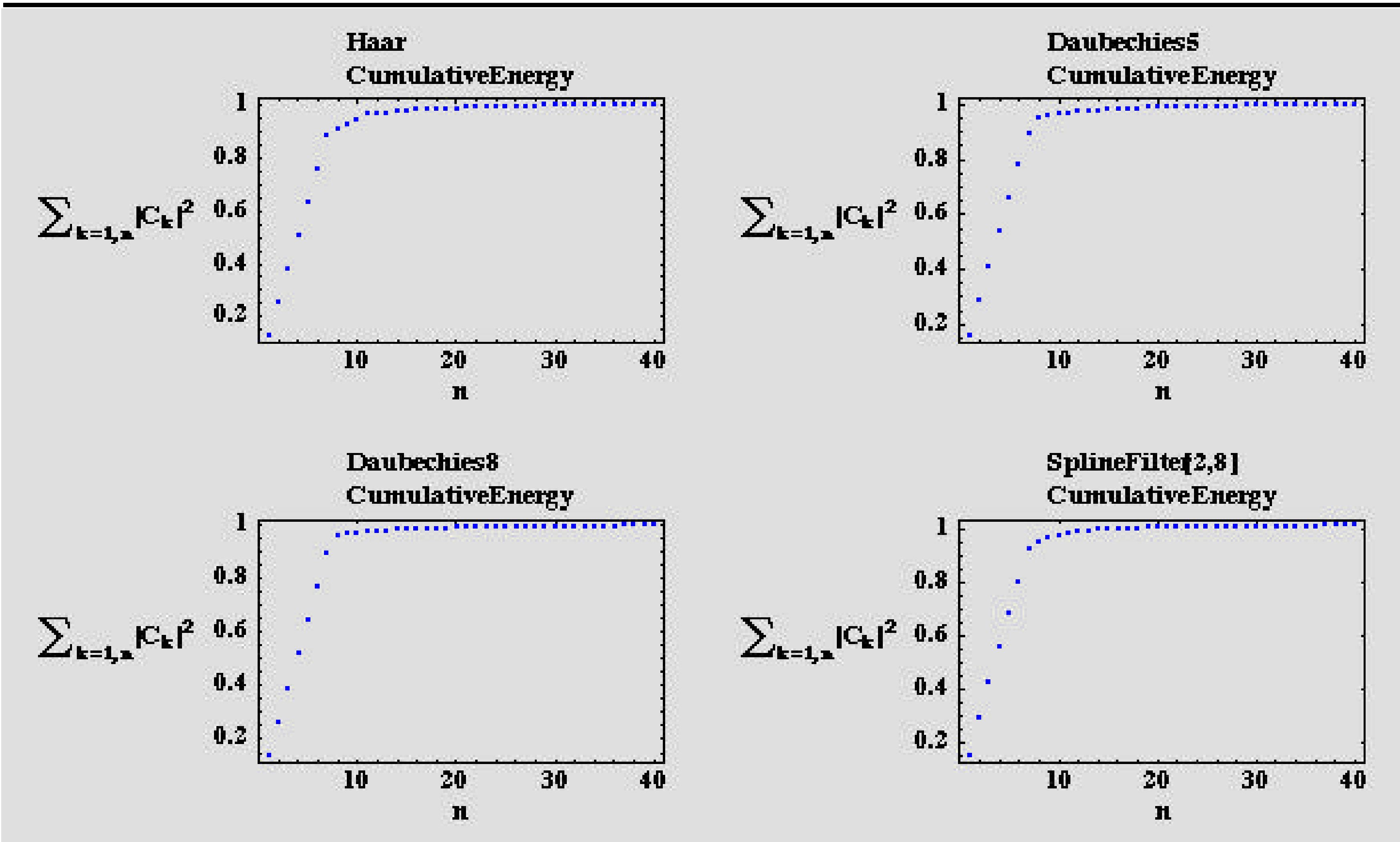

Scalogram: Wavelet Analysis Plot of time vs scale localization of the MRD

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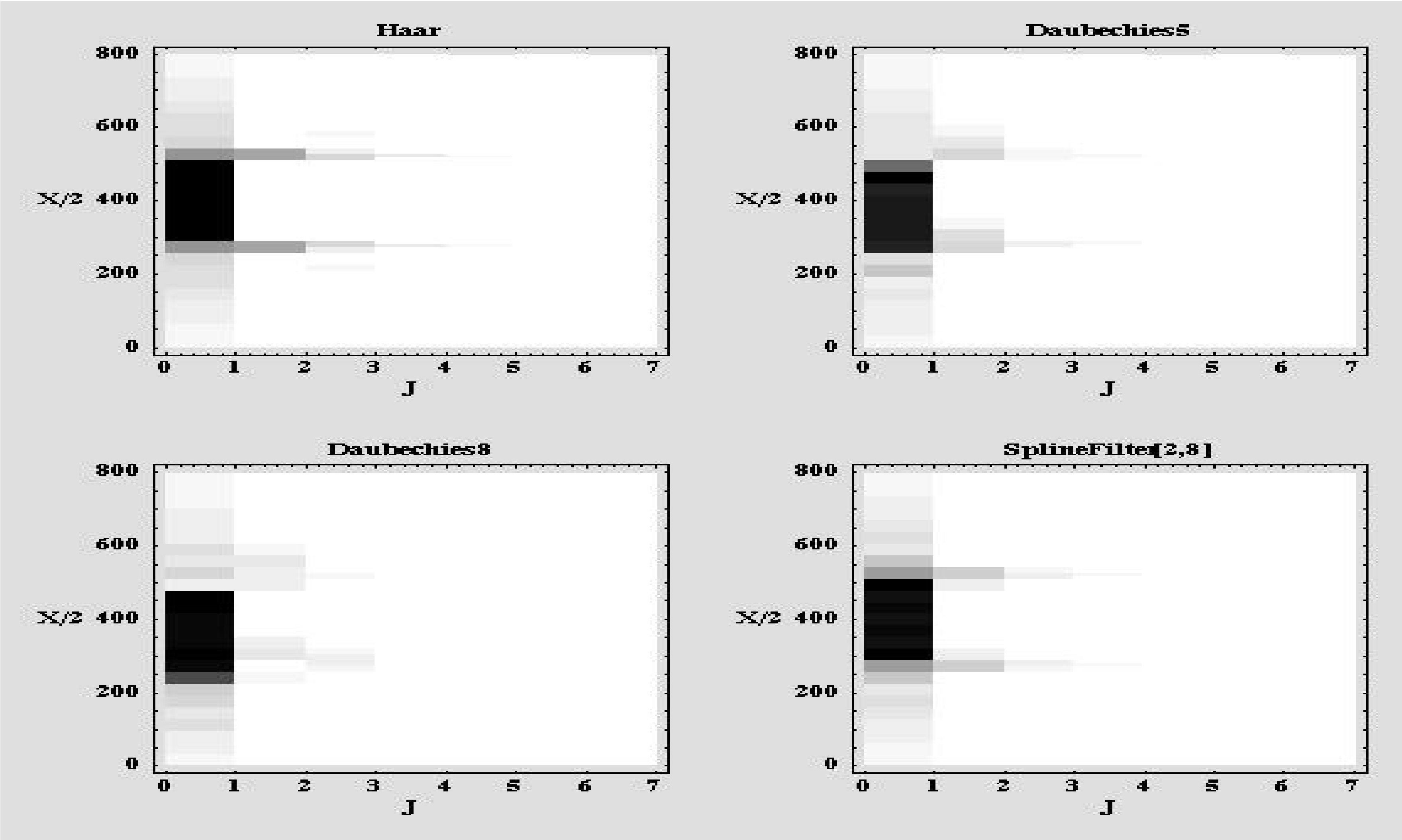

RMS Error vs Number of Largest WLT Coefficients Kept for Four Wavelet MRDs

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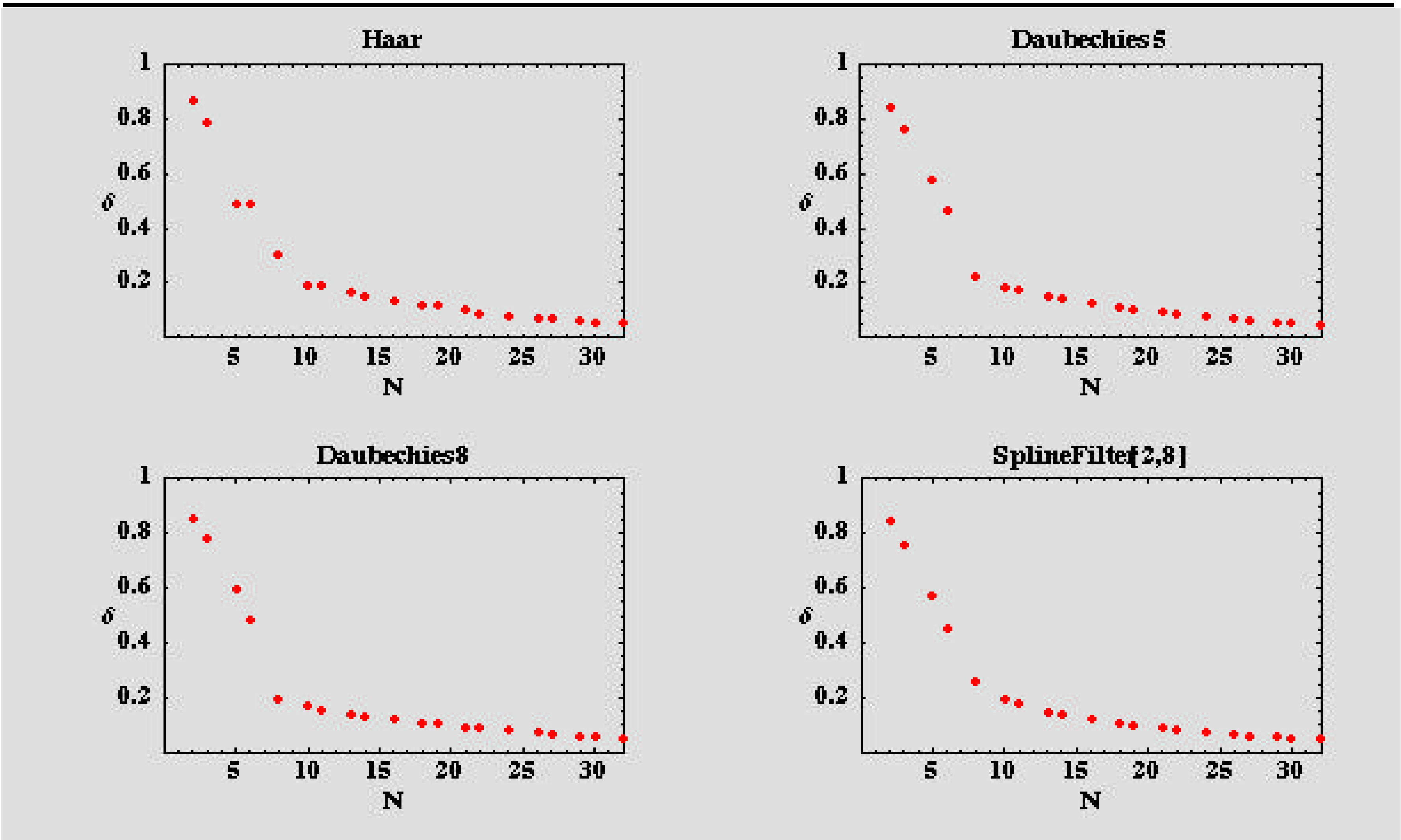

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RMS Error vs MRD Level Kept for Four Different Wavelet Choices

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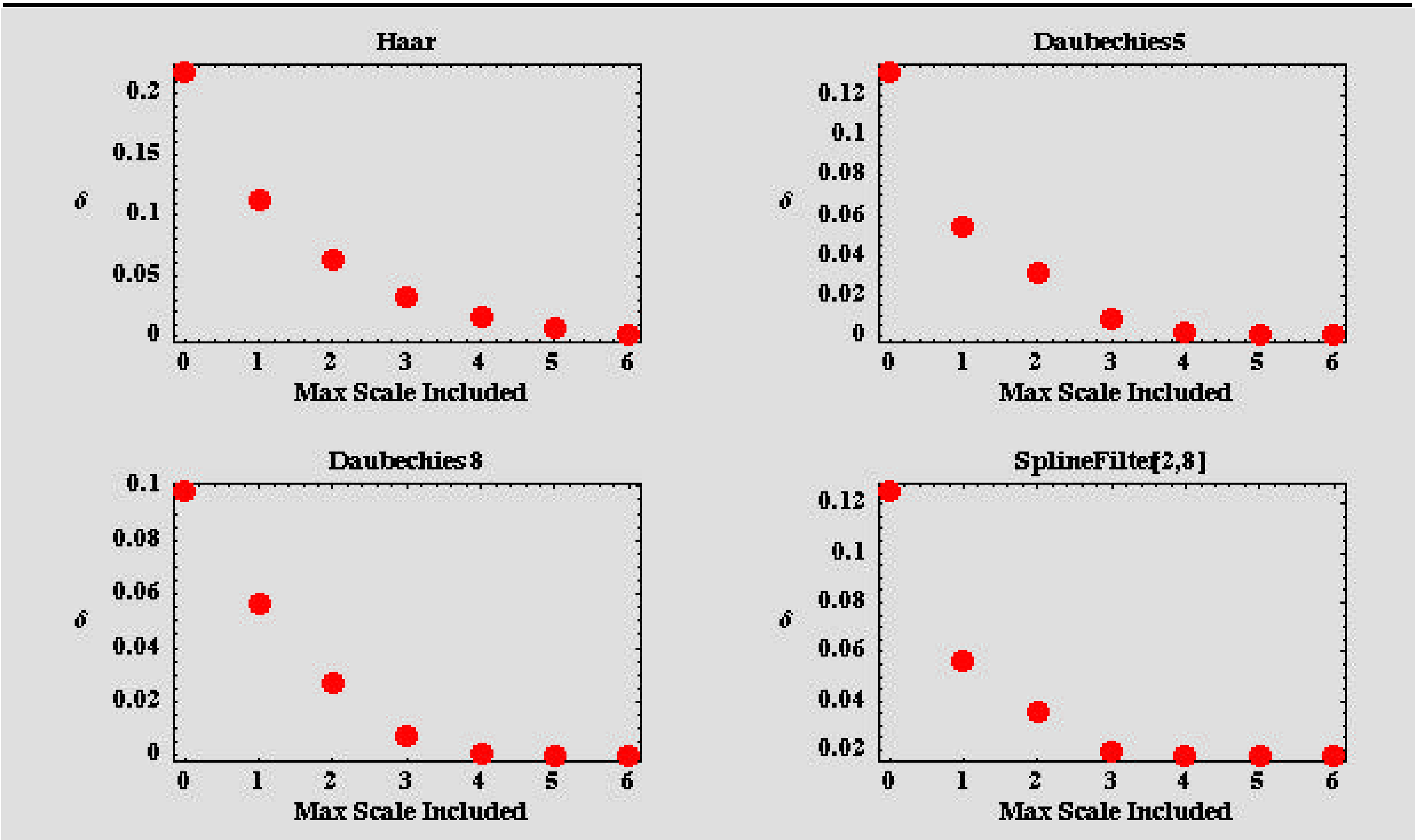

Differences in the Accumulated Energy, Largest Coeffs & RMS Error vs Number of Coeffs Kept

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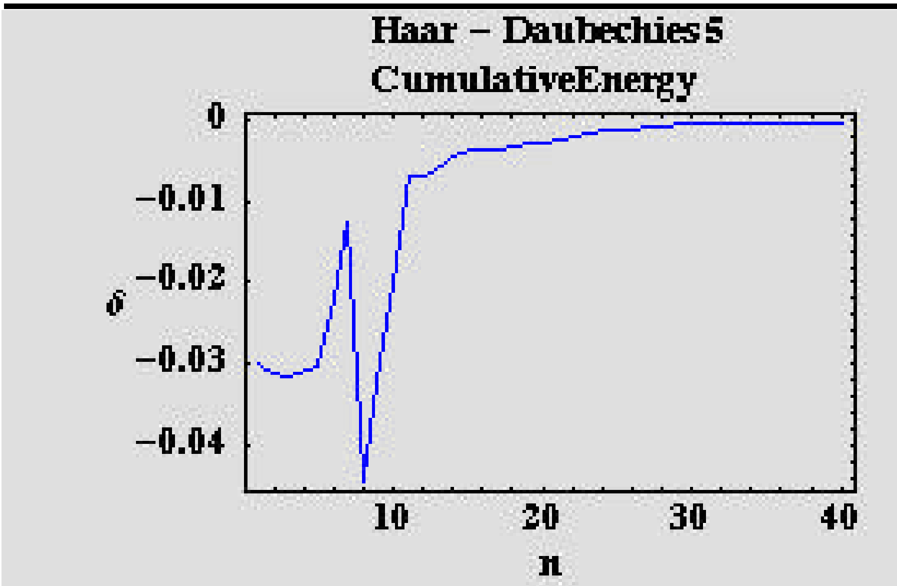
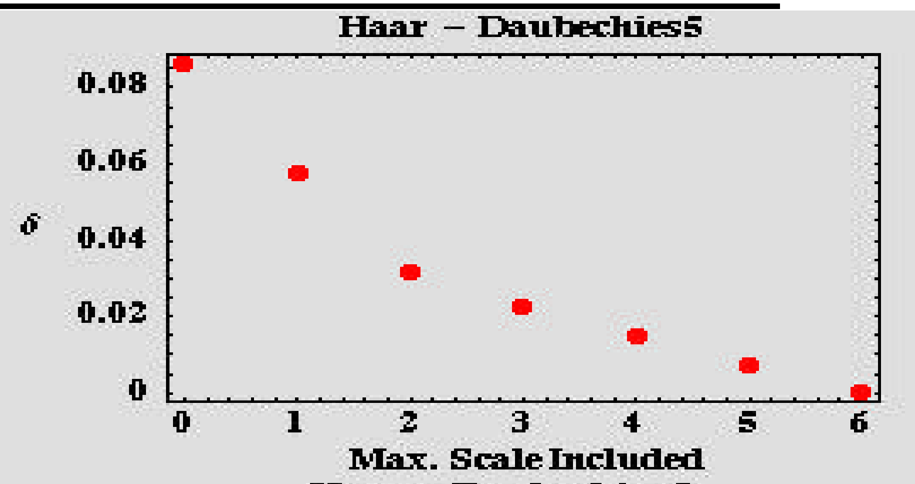
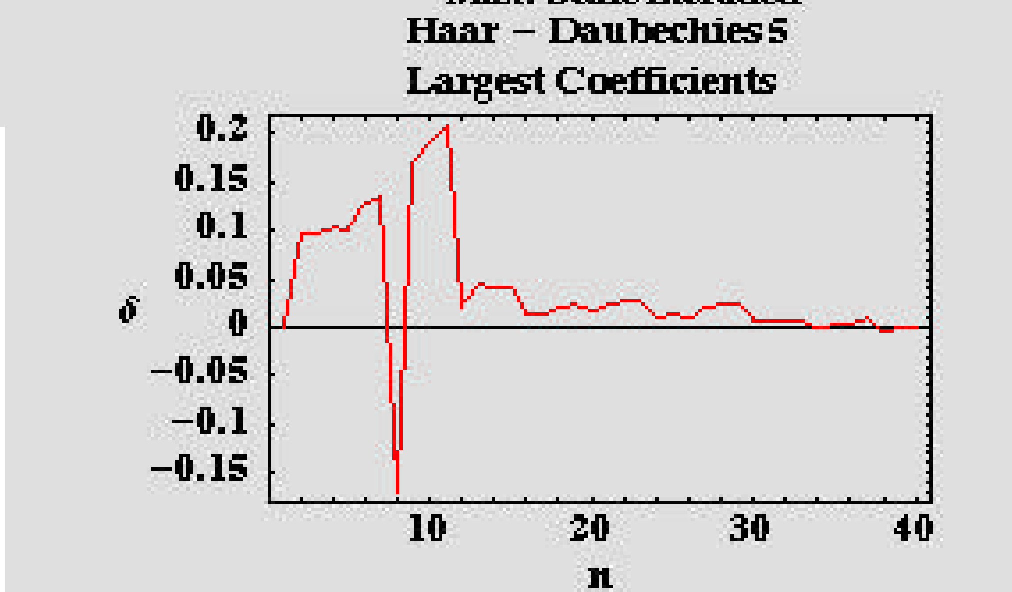
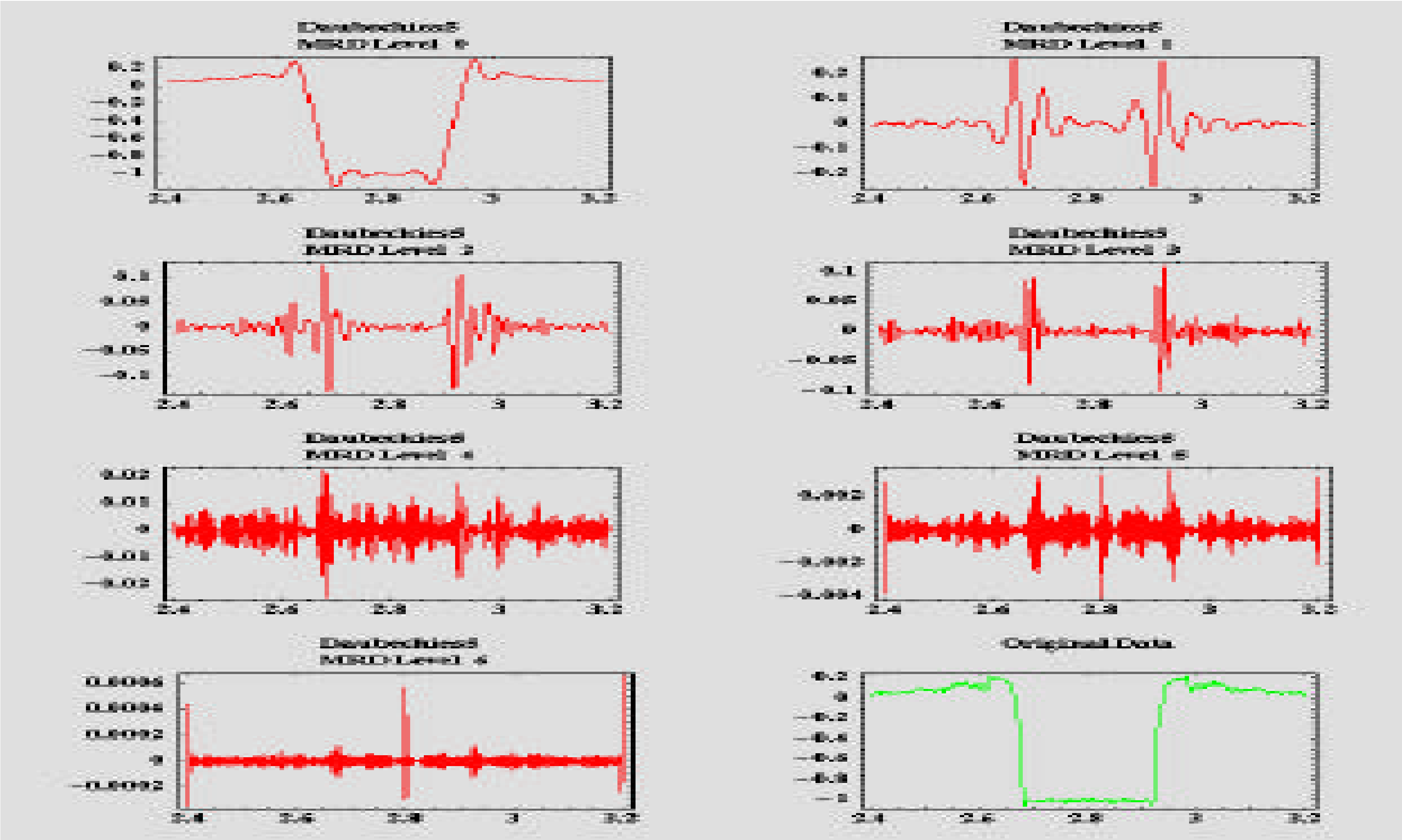


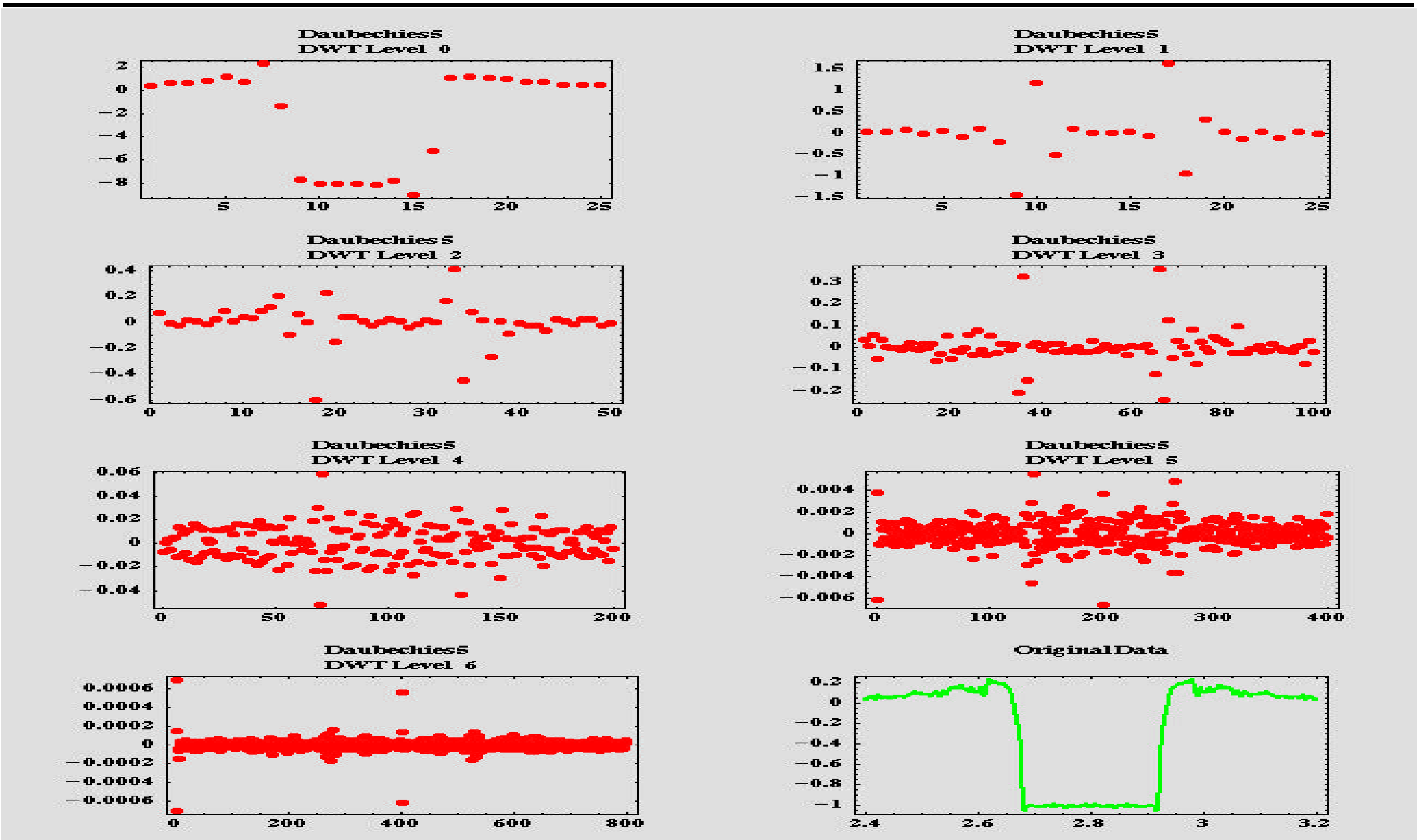




MRD Using Daubechies 5



The Coefficients of the MRD Using Daubechies 5



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Data Compression and Denoising Using (60) Largest Coefficients Thresholding

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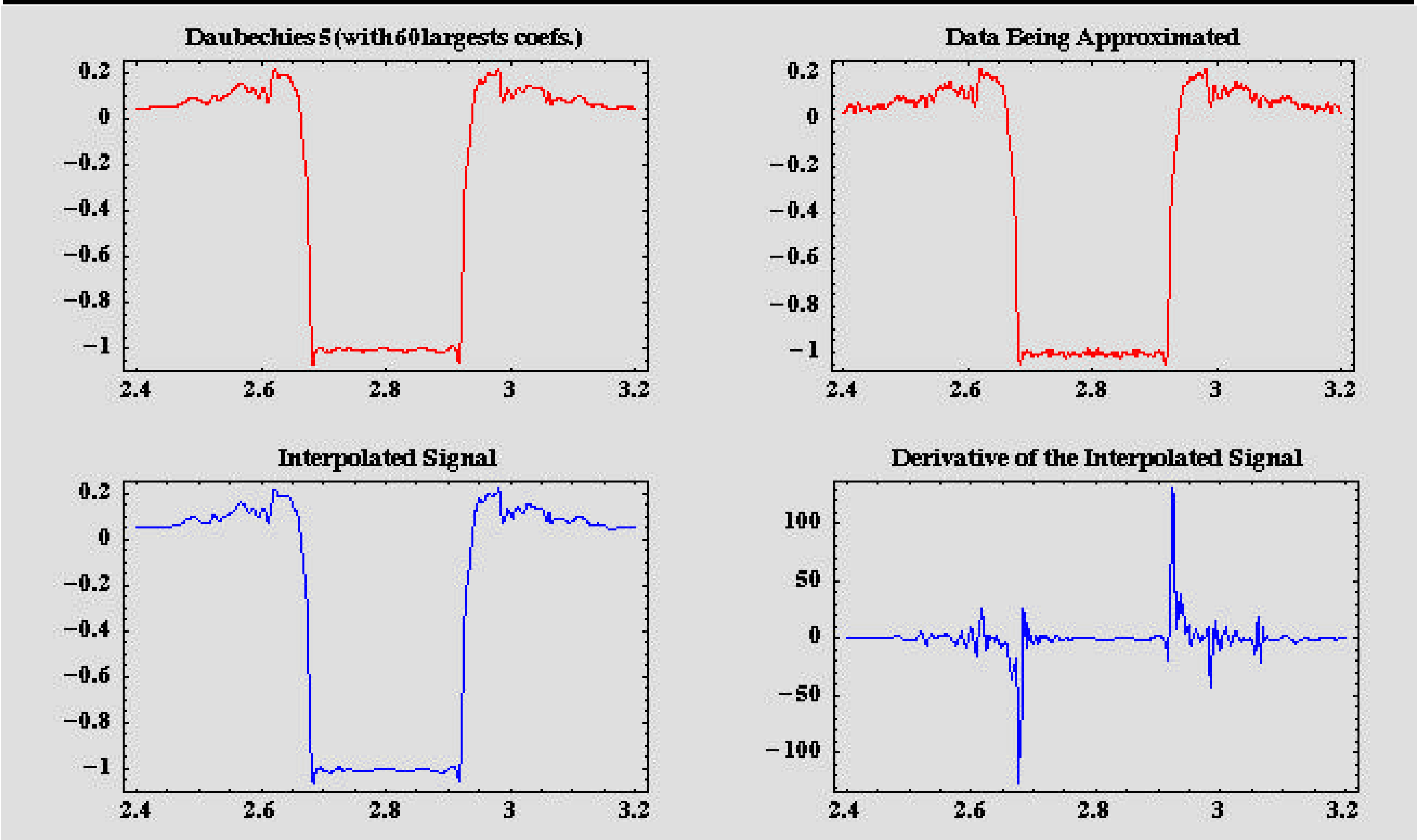

Data Compression and Denoising Using Coeffs Down to 0.1% of Largest Coefficient

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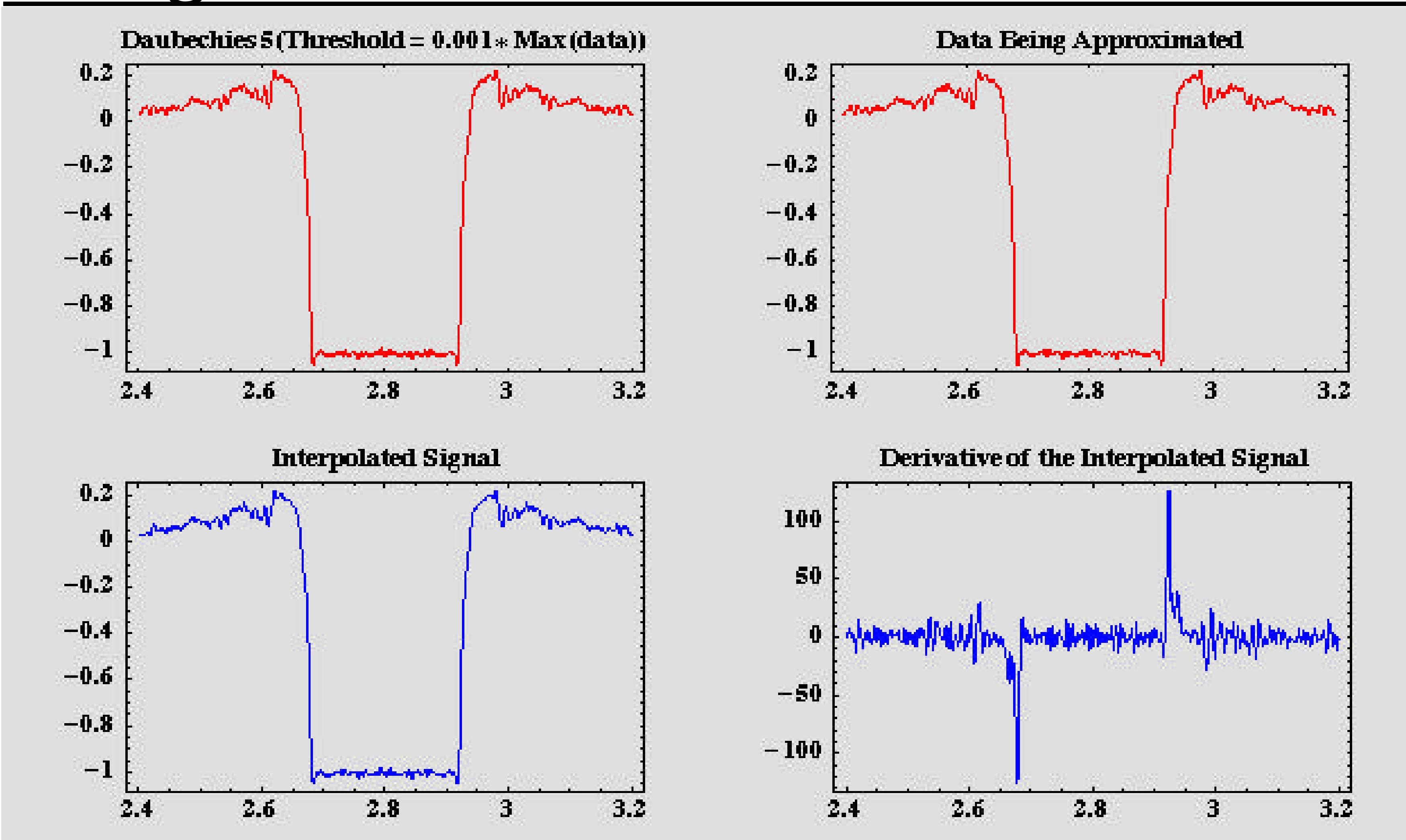

Data Compression and Denoising Using 7 MRD Levels

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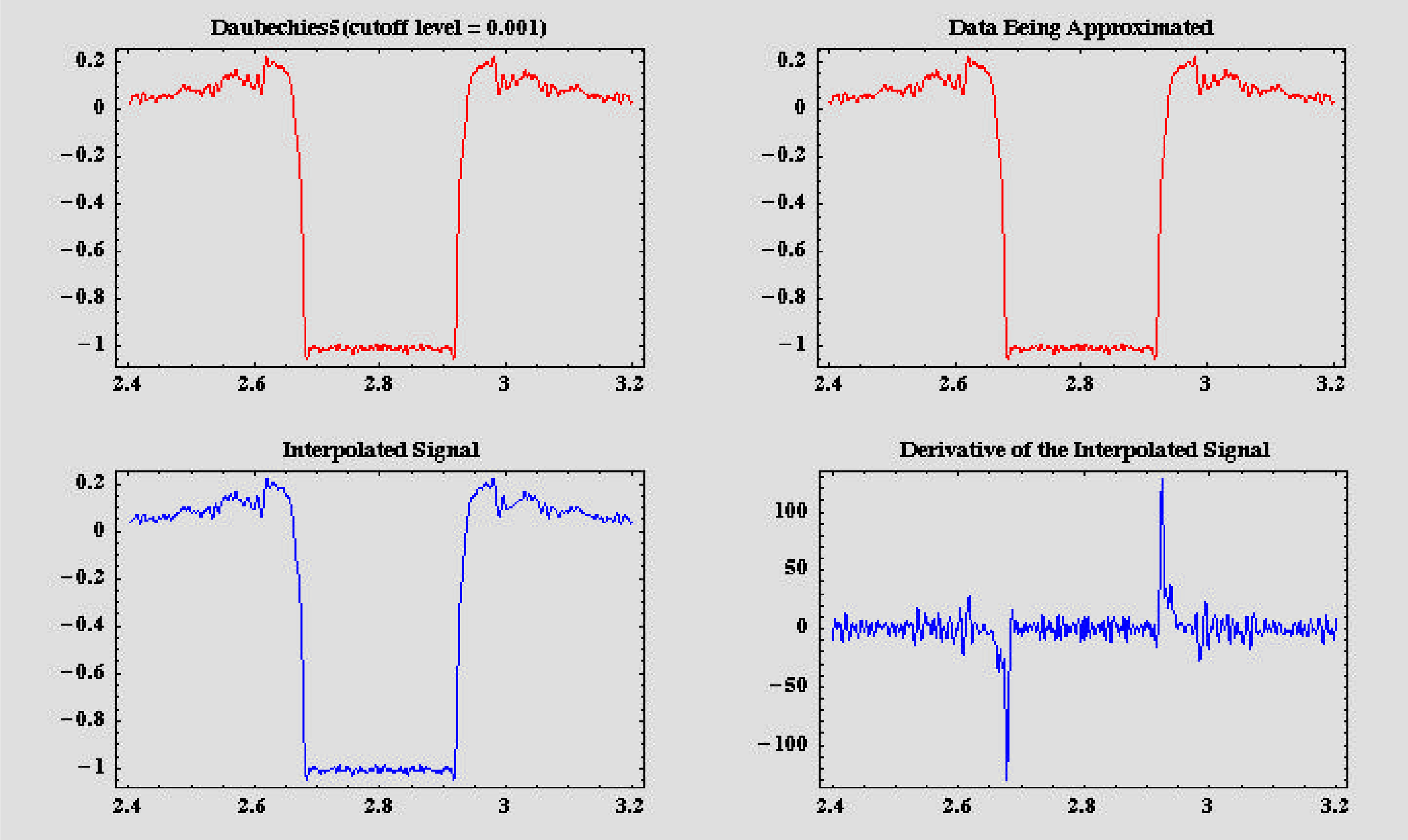

Reconstruction of Z 66 Bolometer Energy & Power Using 10 Largest Wavelet Coefficients

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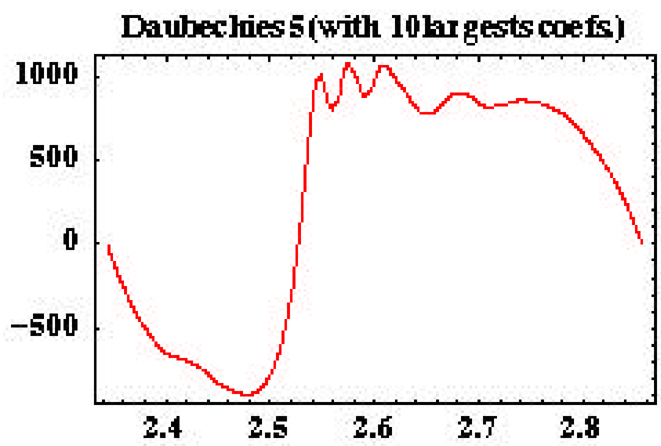
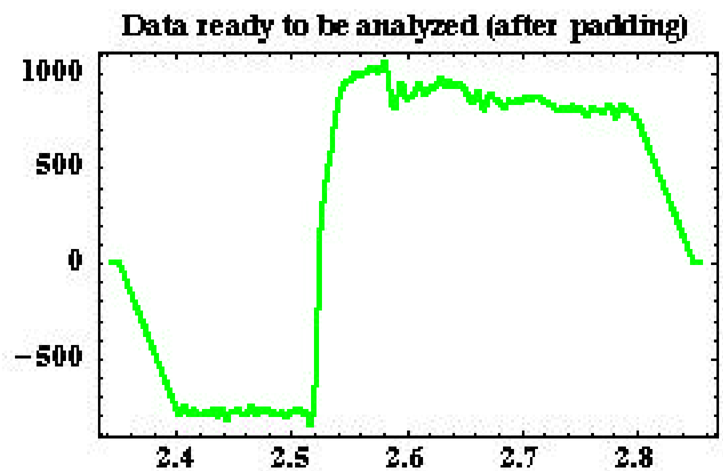
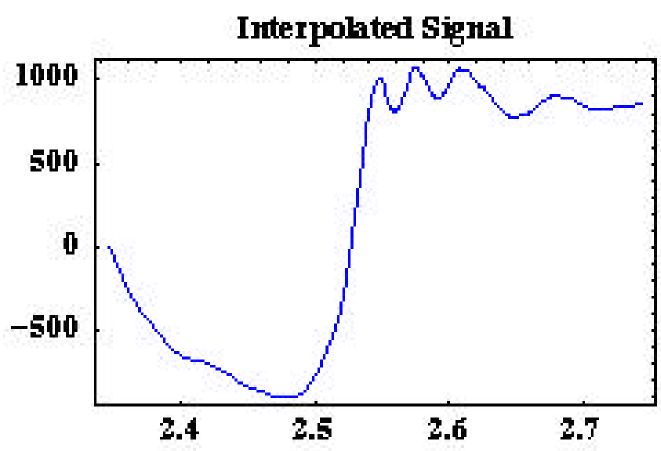
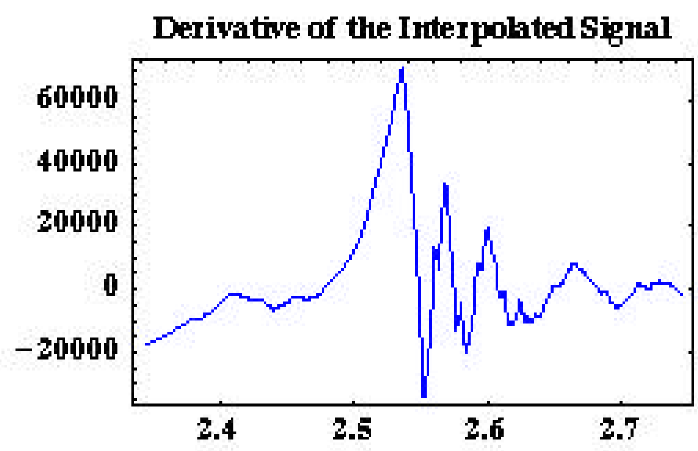
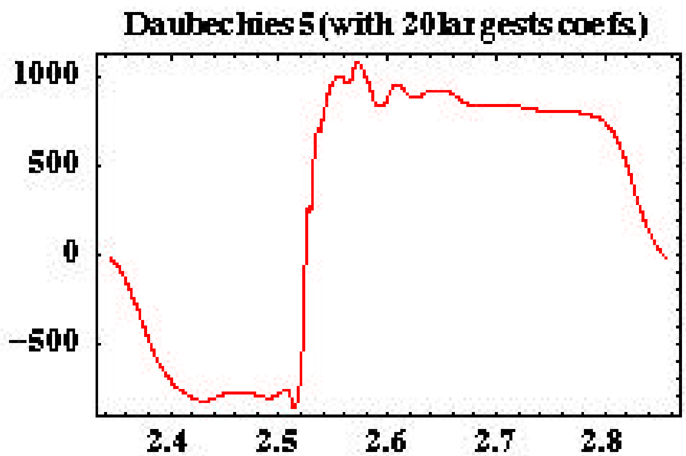


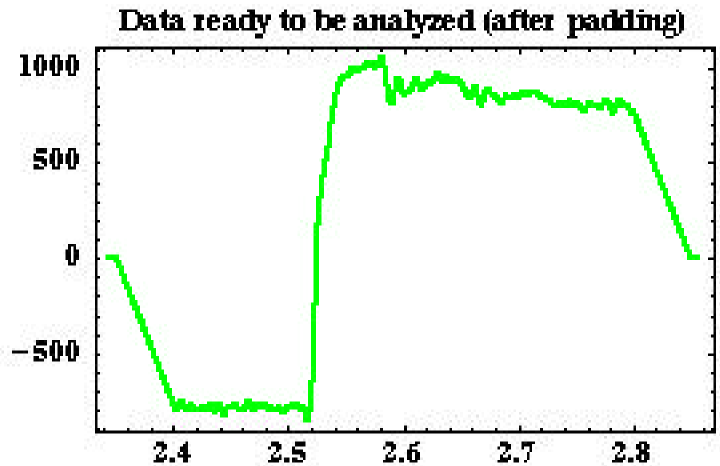


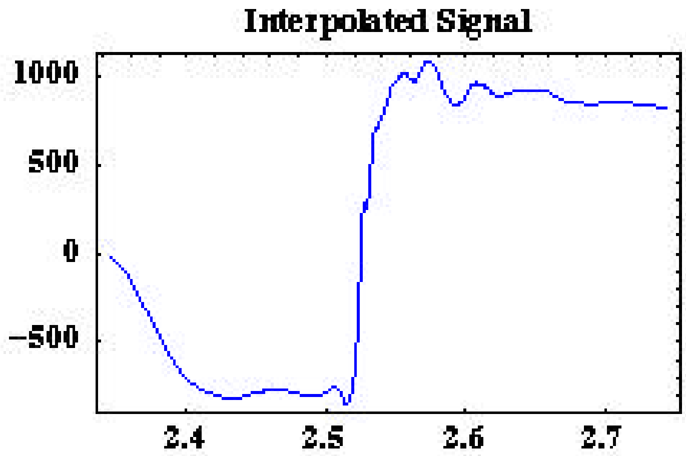


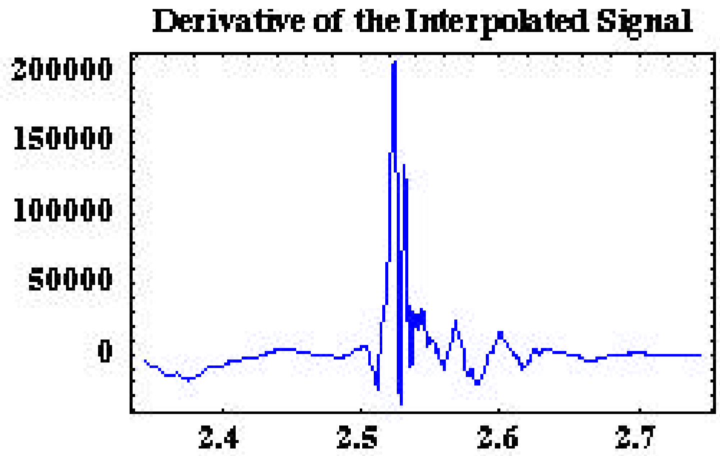


Reconstruction of Z 66 Bolometer Energy & Power Using 20 Largest Wavelet Coefficients









Reconstruction of Z 66 Bolometer Energy & Power Using 30 Largest Wavelet Coefficients

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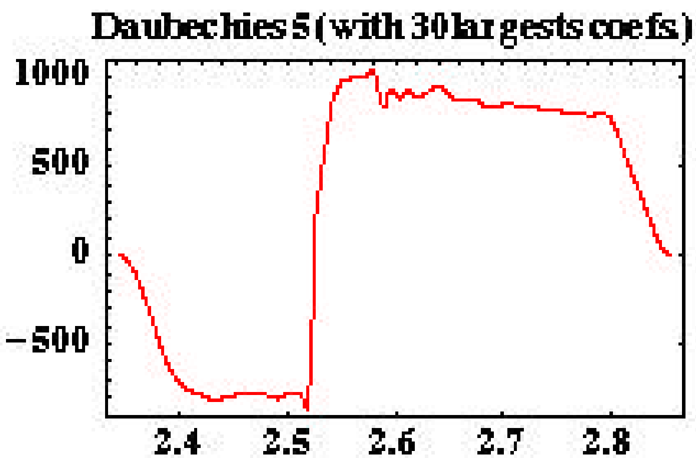

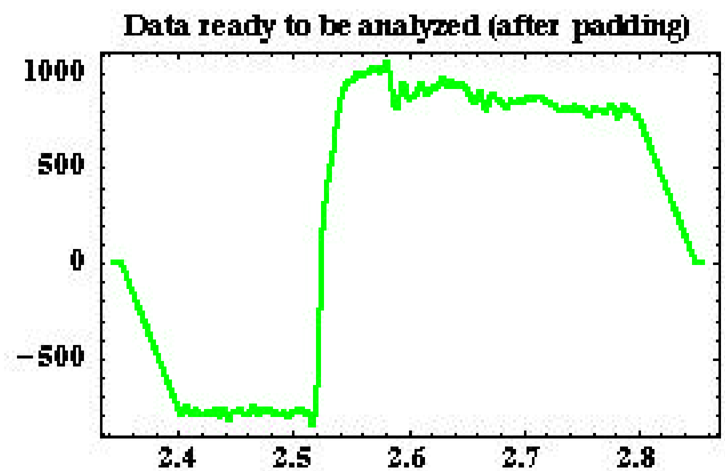

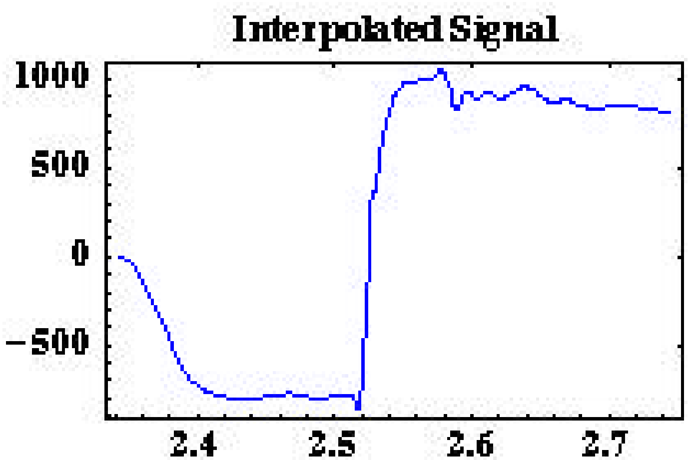

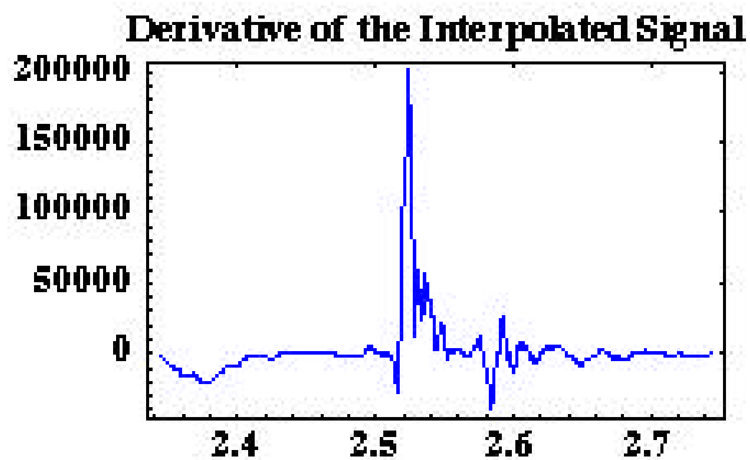

Reconstruction of Z 66 Bolometer Energy & Power Using 40 Largest Wavelet Coefficients

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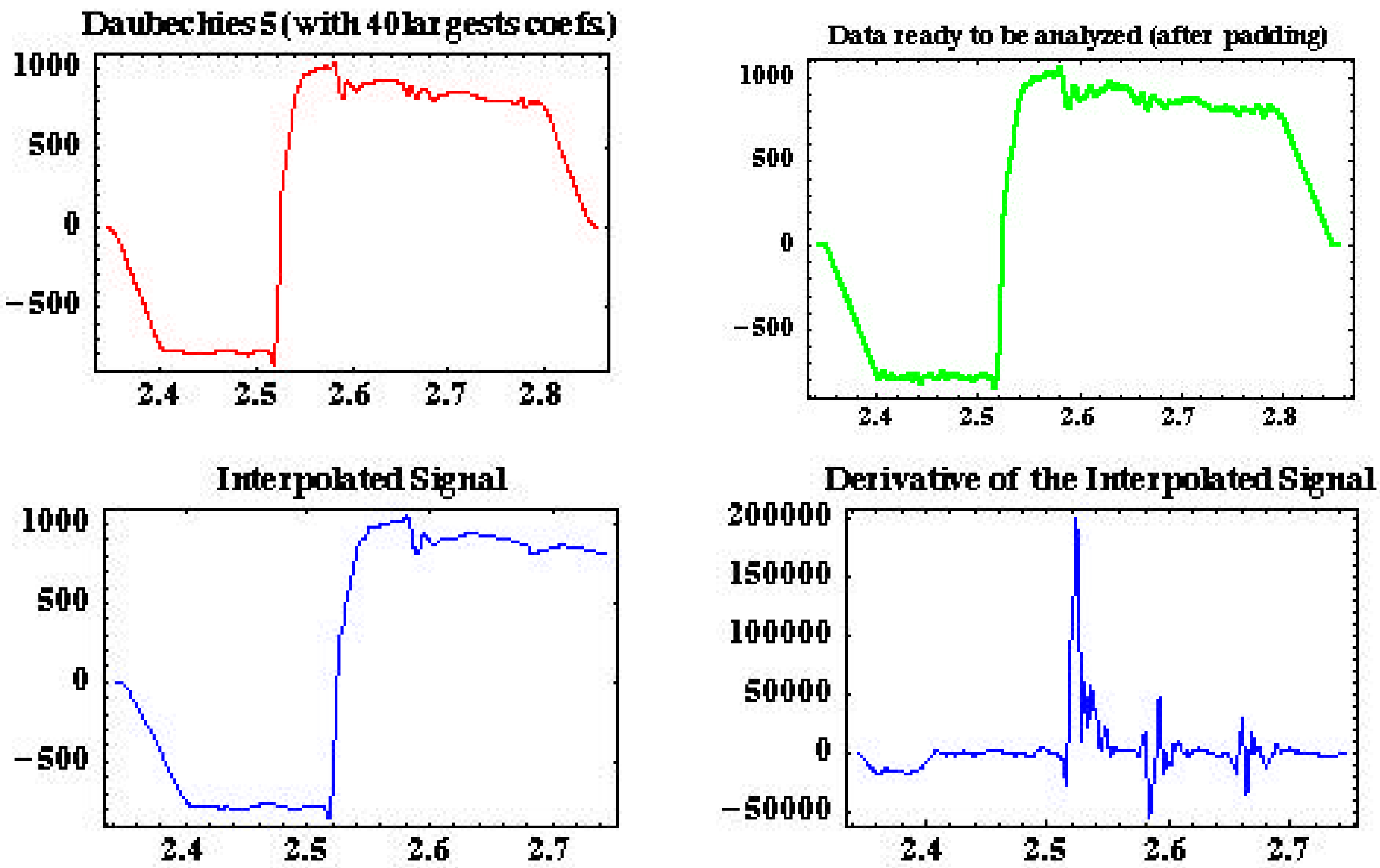

Reconstruction of Z 66 Bolometer Energy & Power Using 60 Largest Wavelet Coefficients

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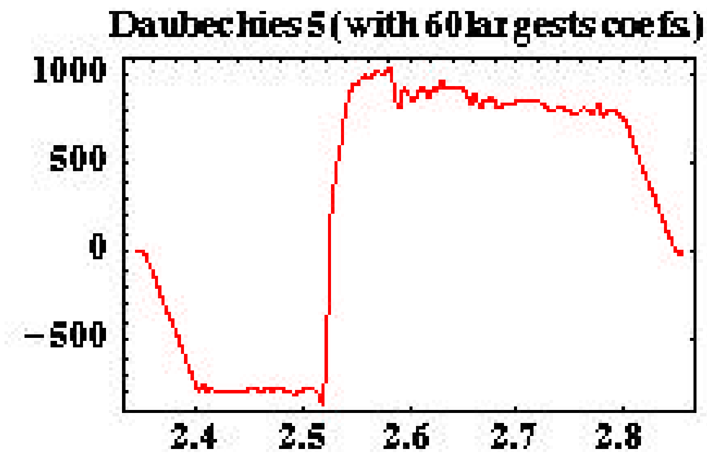

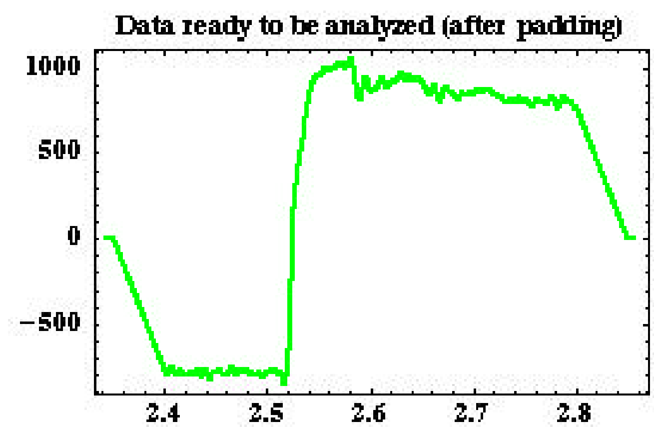

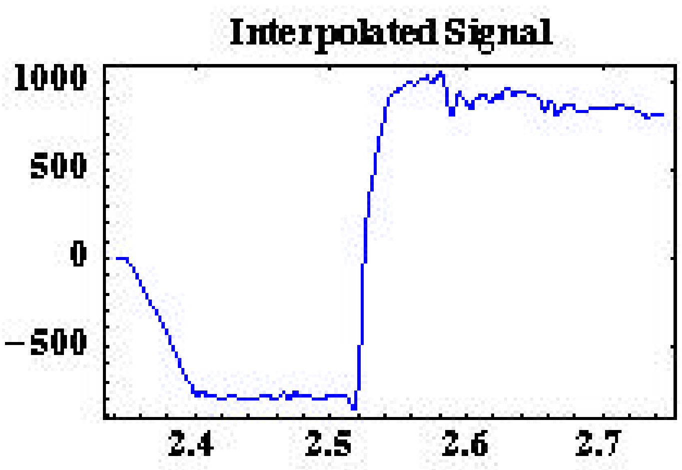

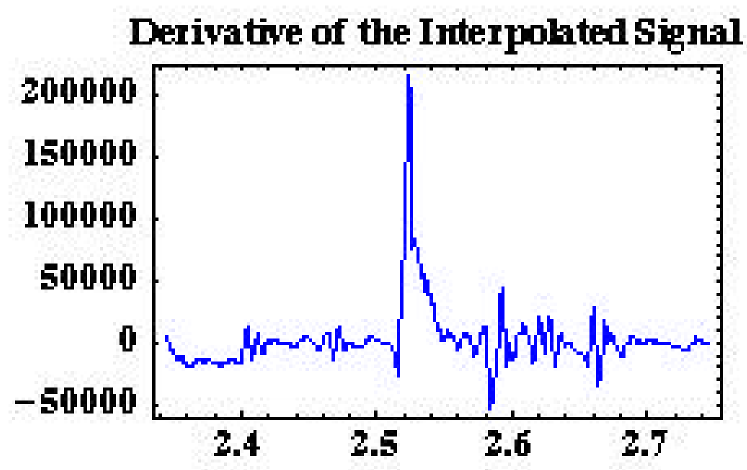

Reconstruction of Z 66 Bolometer Energy Using 100 Largest Wavelet Coefficients

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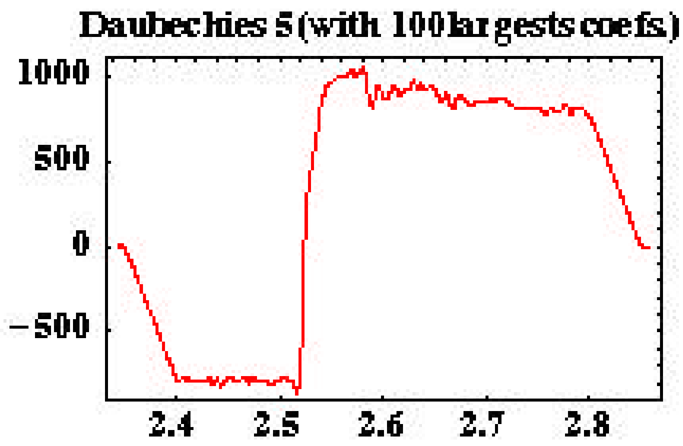
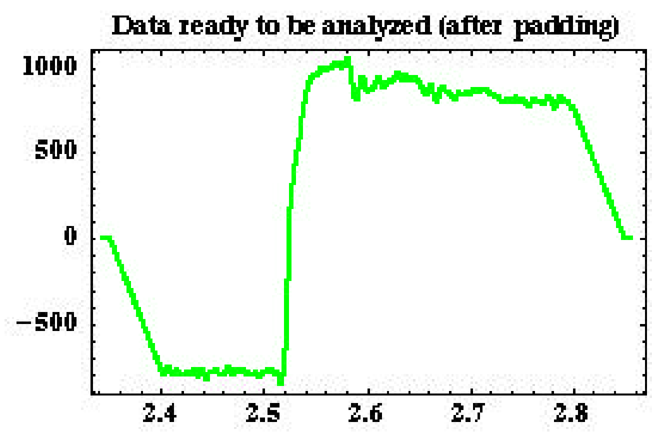
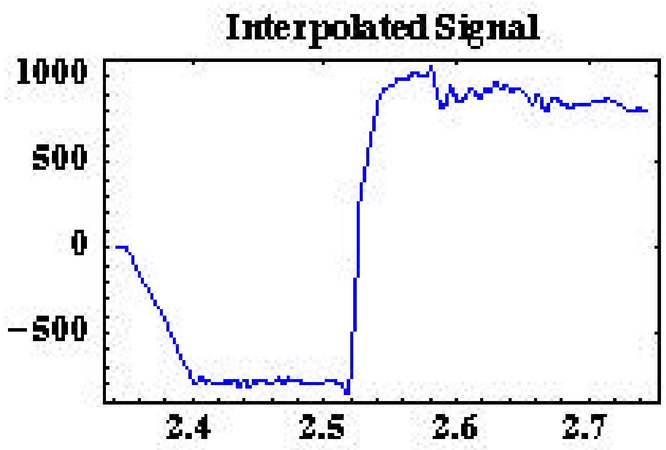
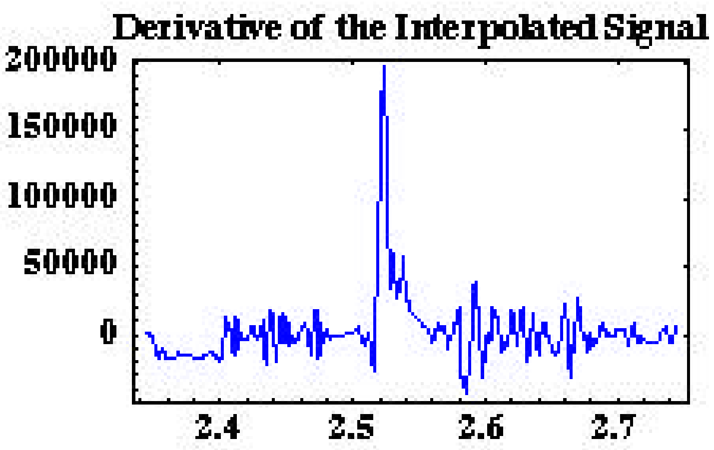
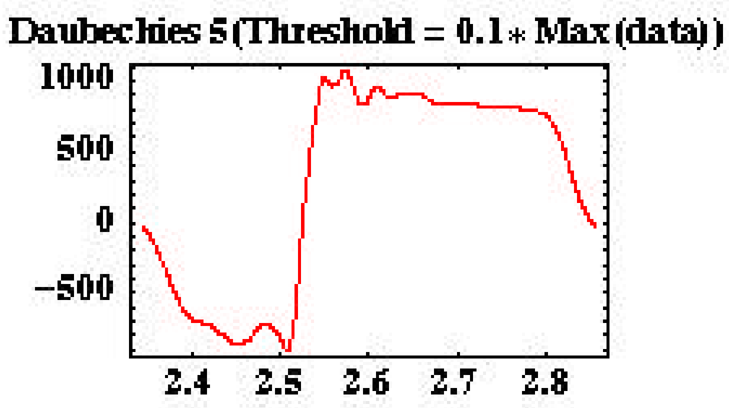


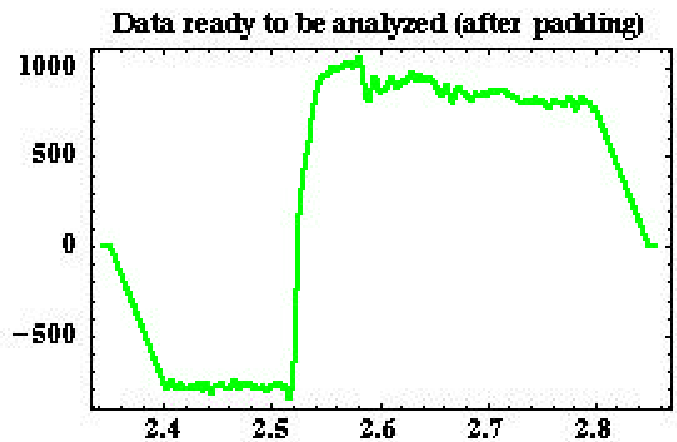


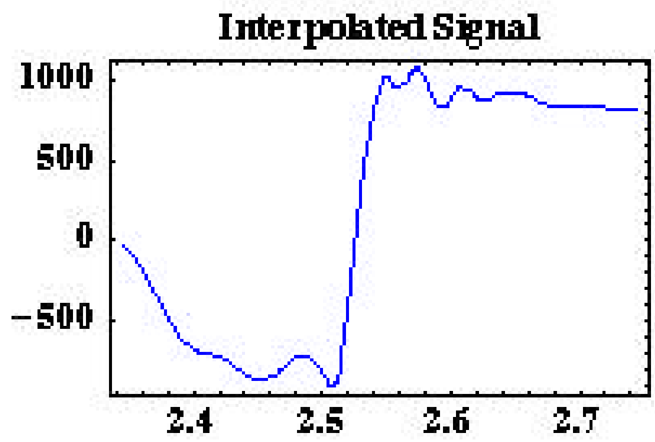


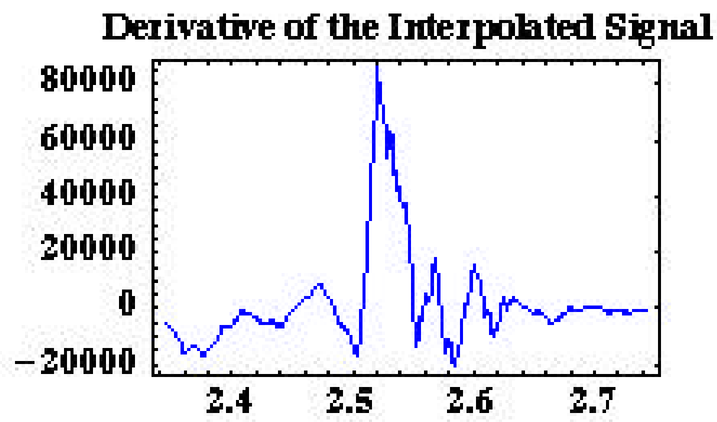


Reconstruction of Z 66 Bolometer Energy & Power Keeping Up to 10% of Maximum Amplitude









Reconstruction of Z 66 Bolometer Energy & Power Keeping Up to 1% of Maximum Amplitude

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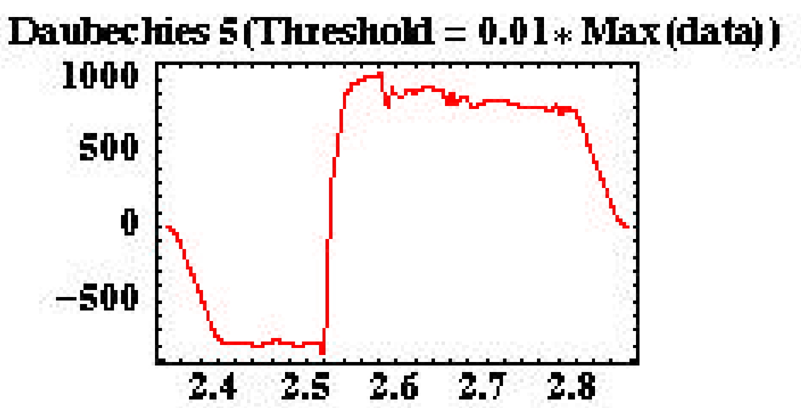
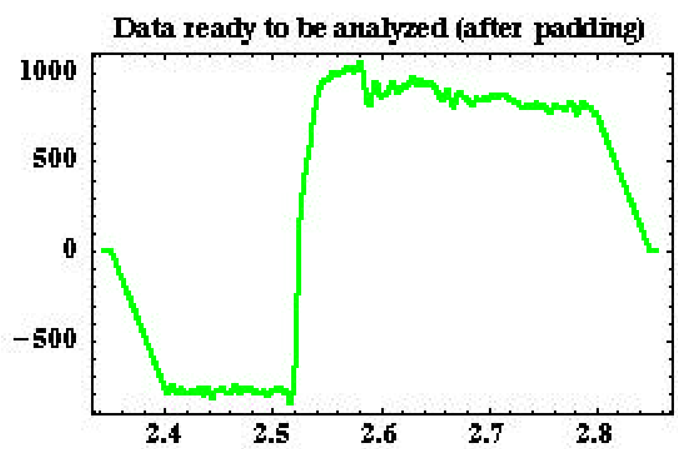
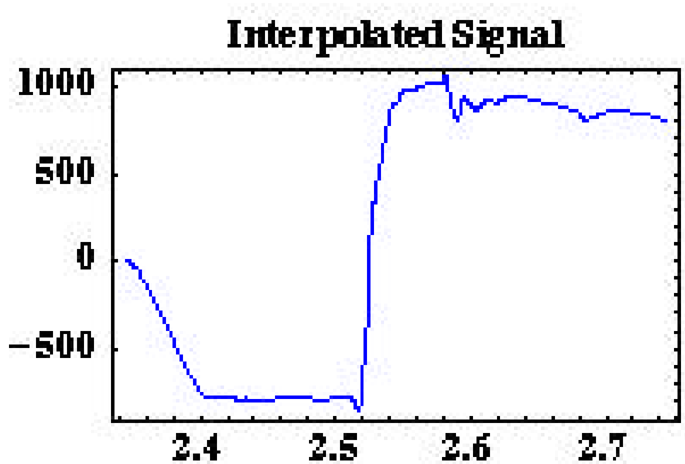
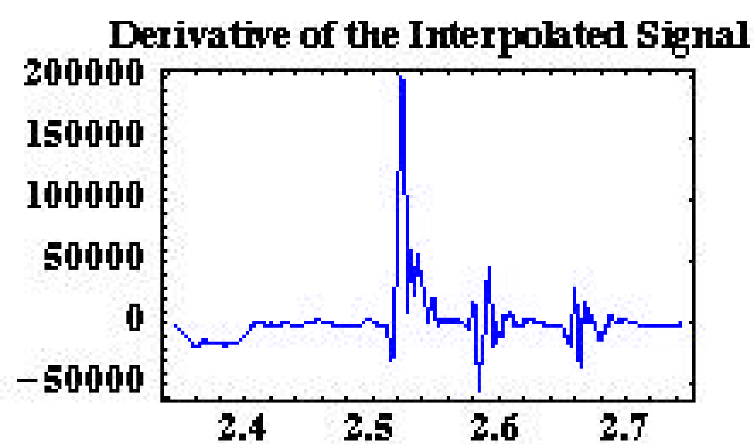
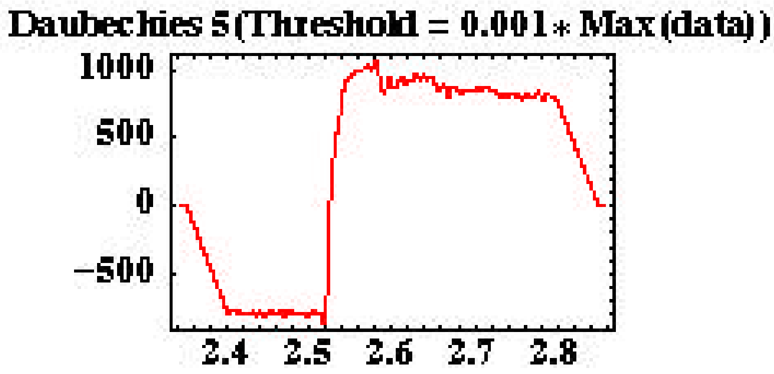


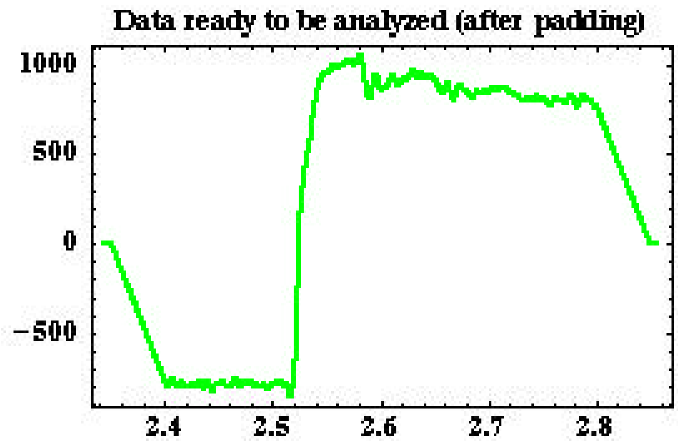


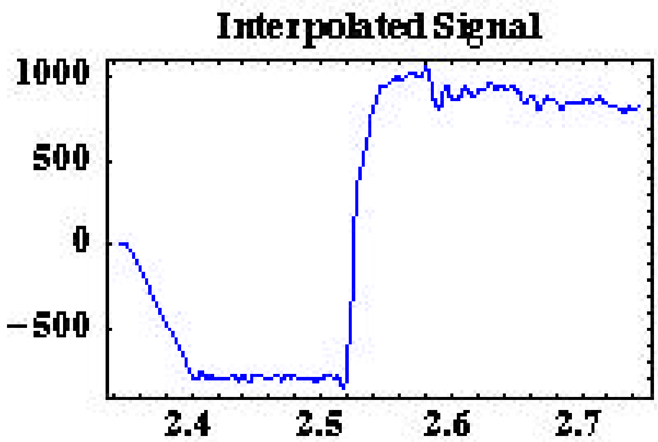


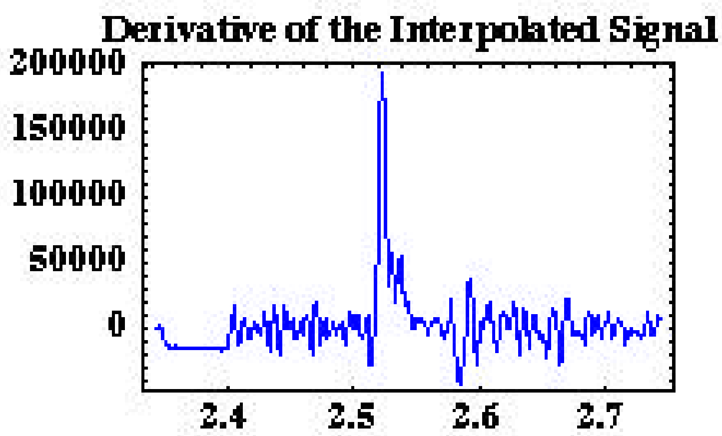


Reconstruction of Z 66 Bolometer Energy & Power Keeping Up to 0.1% of Maximum Amplitude

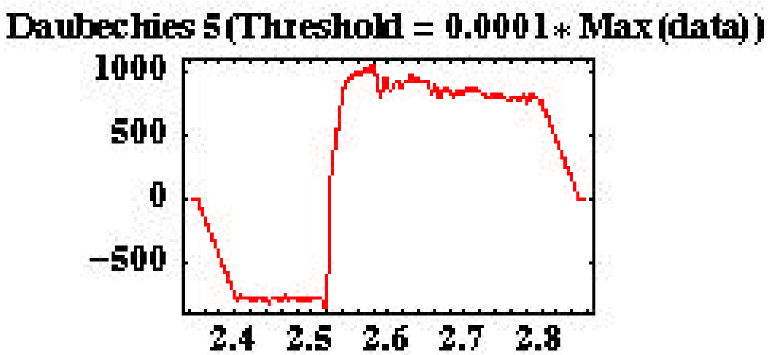


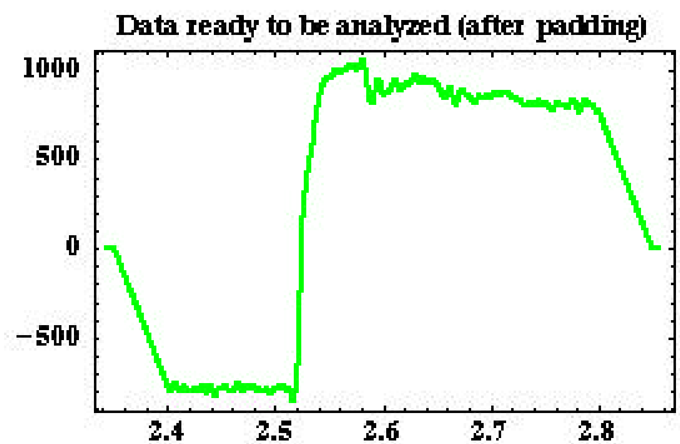


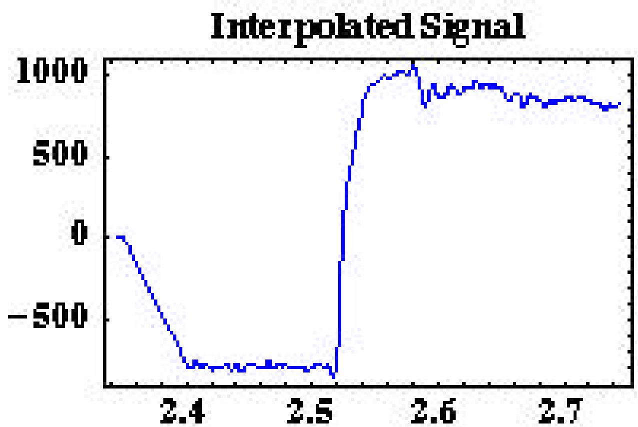


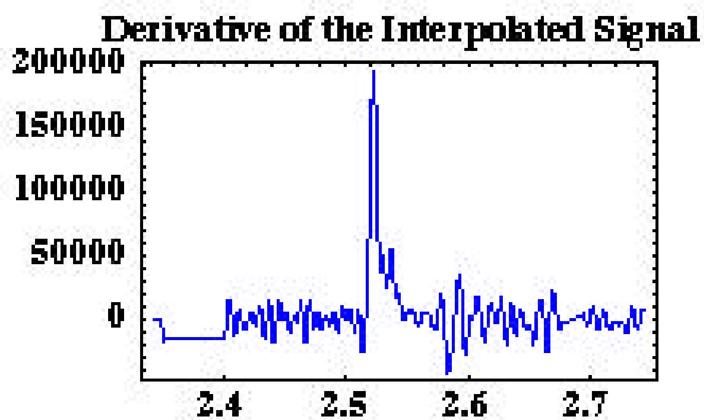


Reconstruction of Z 66 Bolometer Energy & Power Keeping Up to 0.01% of Maximum Amplitude









Empirical Conclusions:

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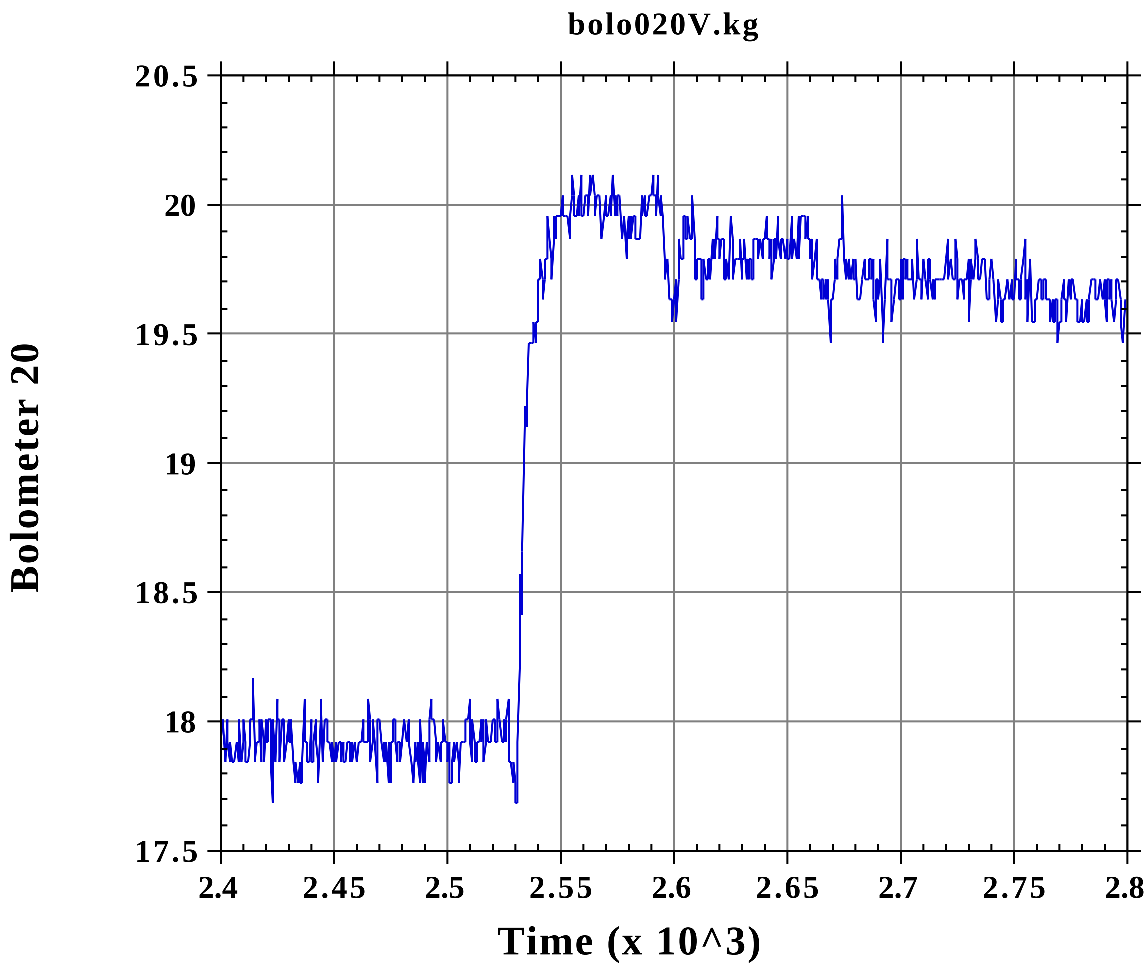
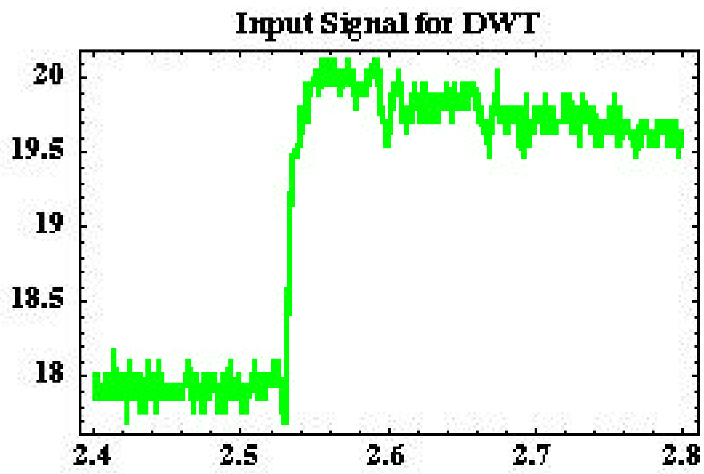
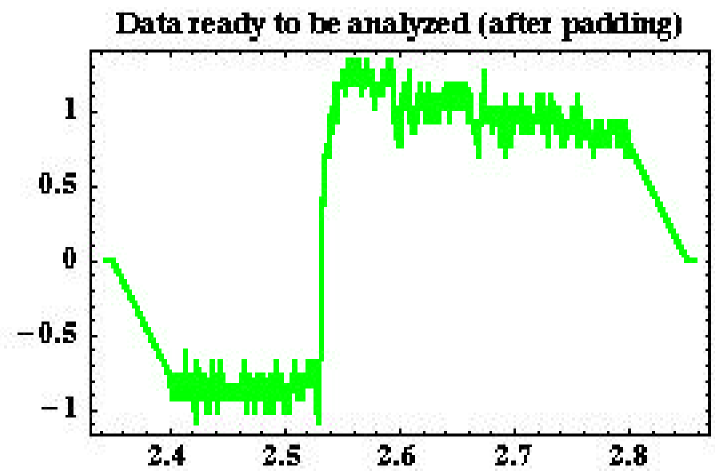
- Need to keep coefficients whose size is at least 1% of the largest coefficient's size for good power curve extraction from the energy lineouts.
- Need at least the first 100 largest wavelet coefficients before good power spikes can be extracted from the energy data.
- Need at least the first 5 levels (out of 9) of MRD in order extract good power signals from the energy data. This is not a very promising strategy...

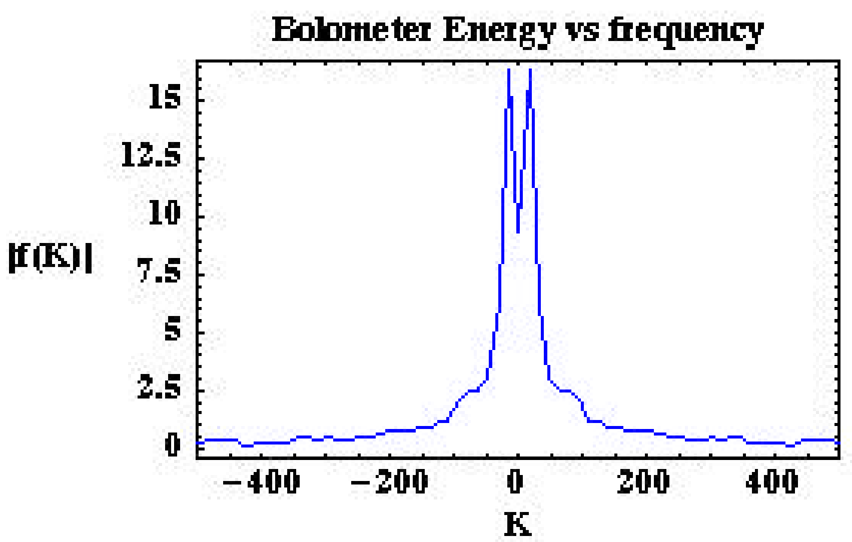
Noisier Bolometer Energy 020V



Bolometer Energy vs Time Signal







The MRD Coefficients for A Noisy Bolometer Energy Lineout

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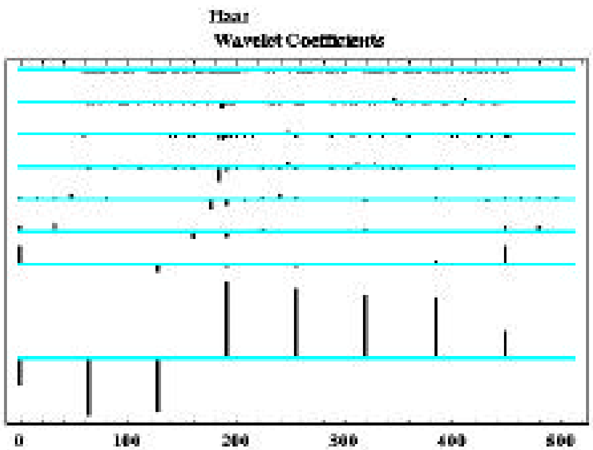
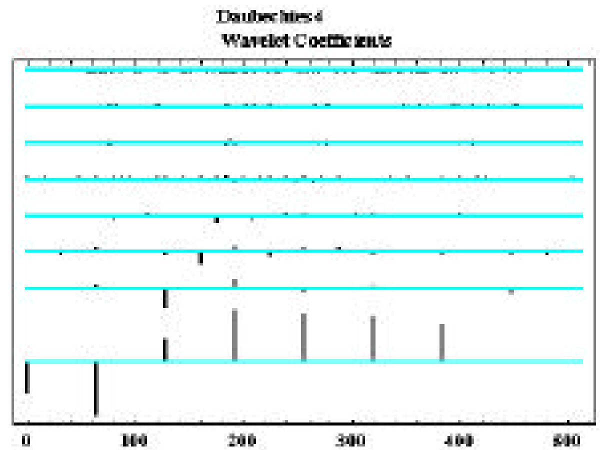
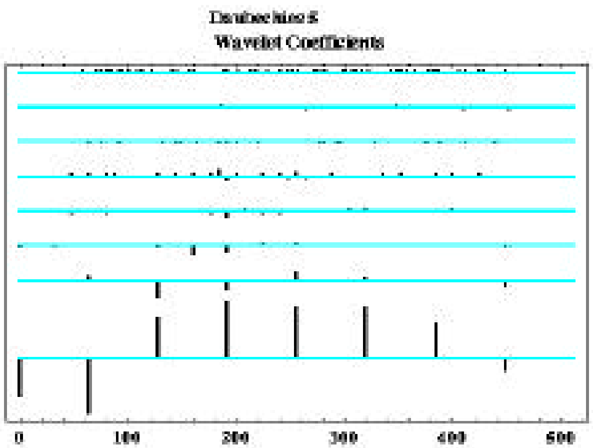
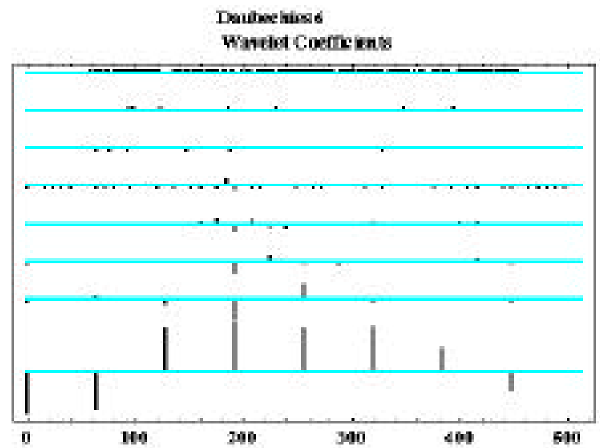


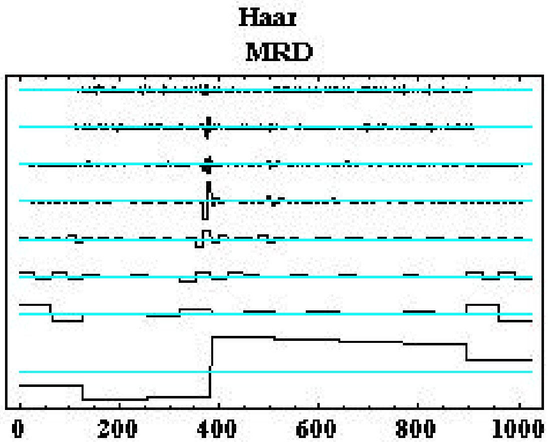


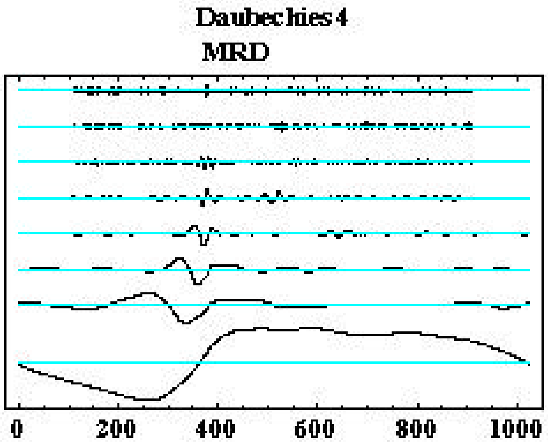


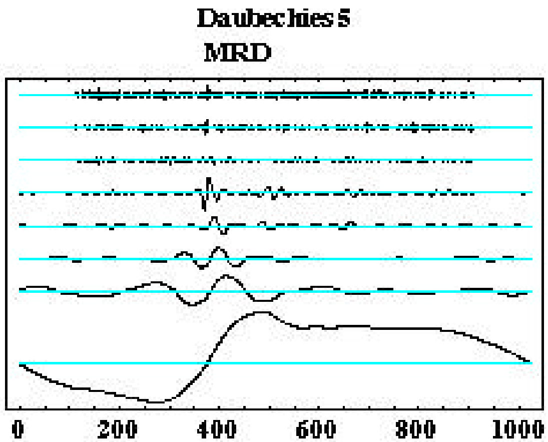


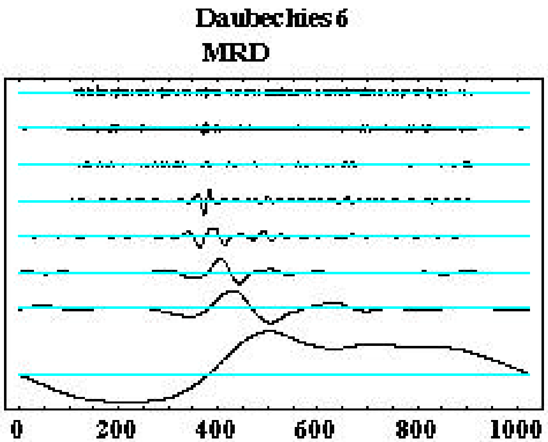
MRD for Noisy Bolometer Energy via 4 Different Wavelet Families

Polymath Research Inc.

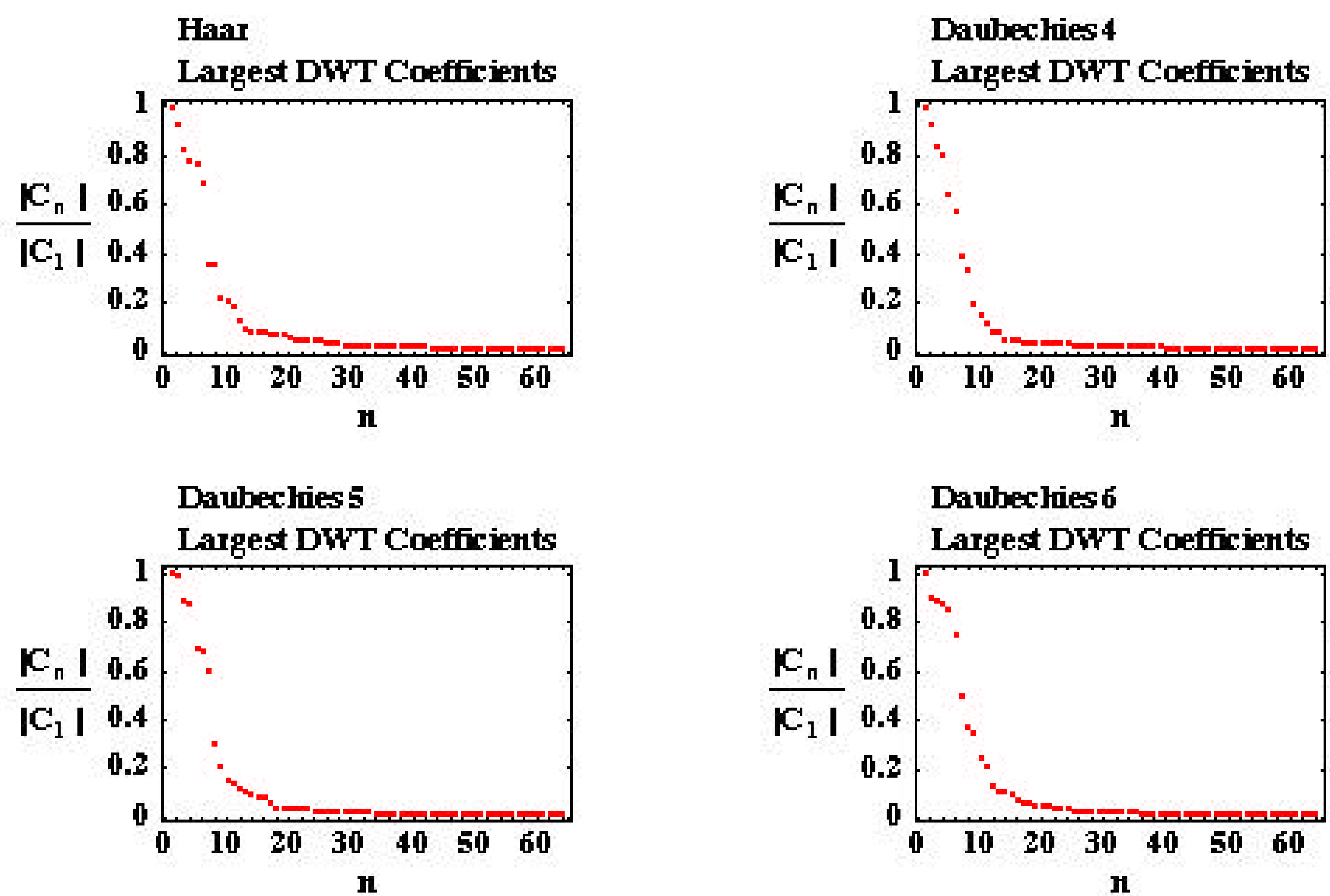






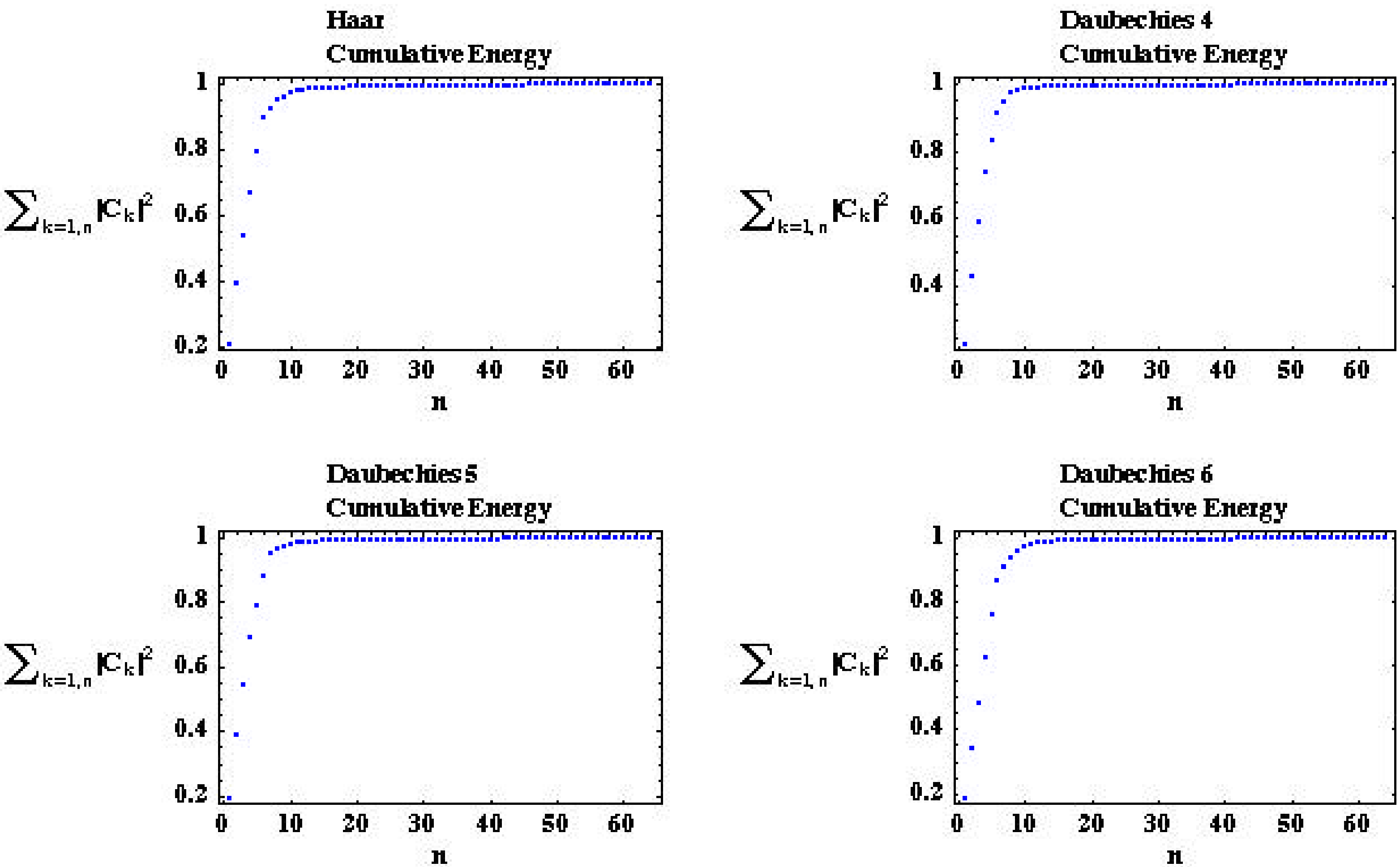


The Largest Coefficients' Decay Rate of Noisy Bolometer Energy



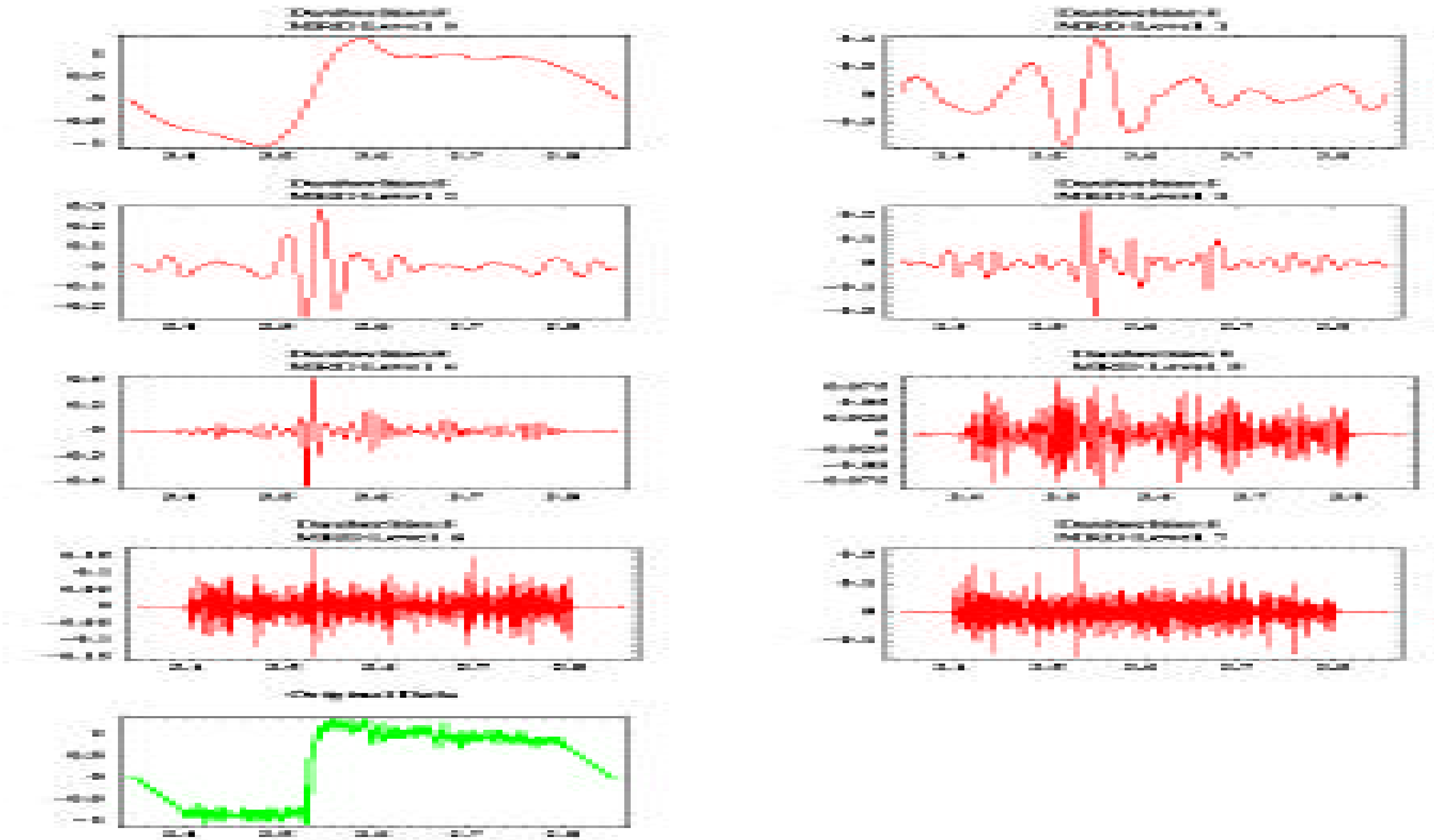
The Energy Accumulation Rate in the Largest Coefficients

Polymath Research Inc.



The MRD Using Daubechies 5 of Noisy Bolometer Energy vs Time

Polymath Research Inc.



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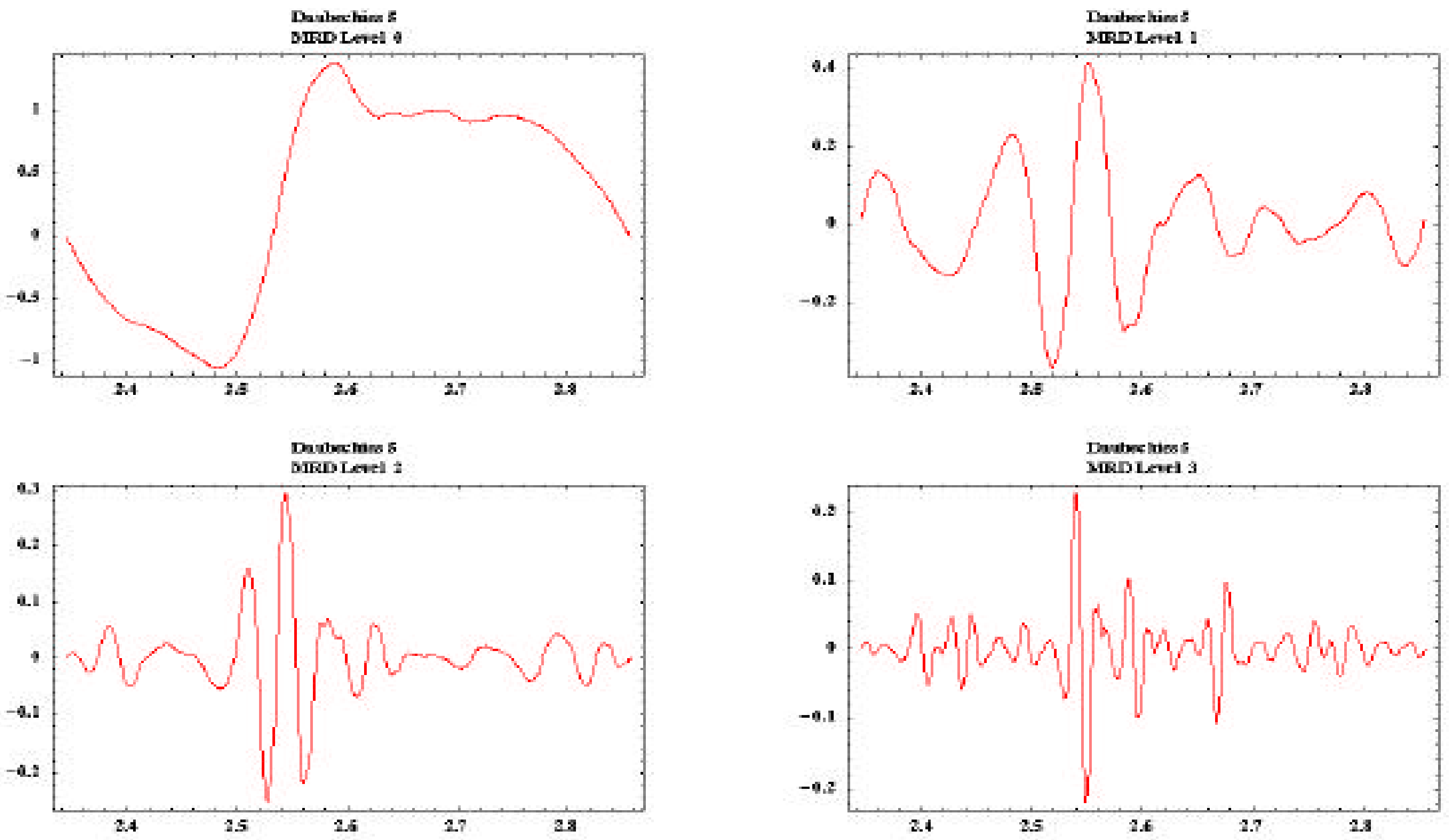
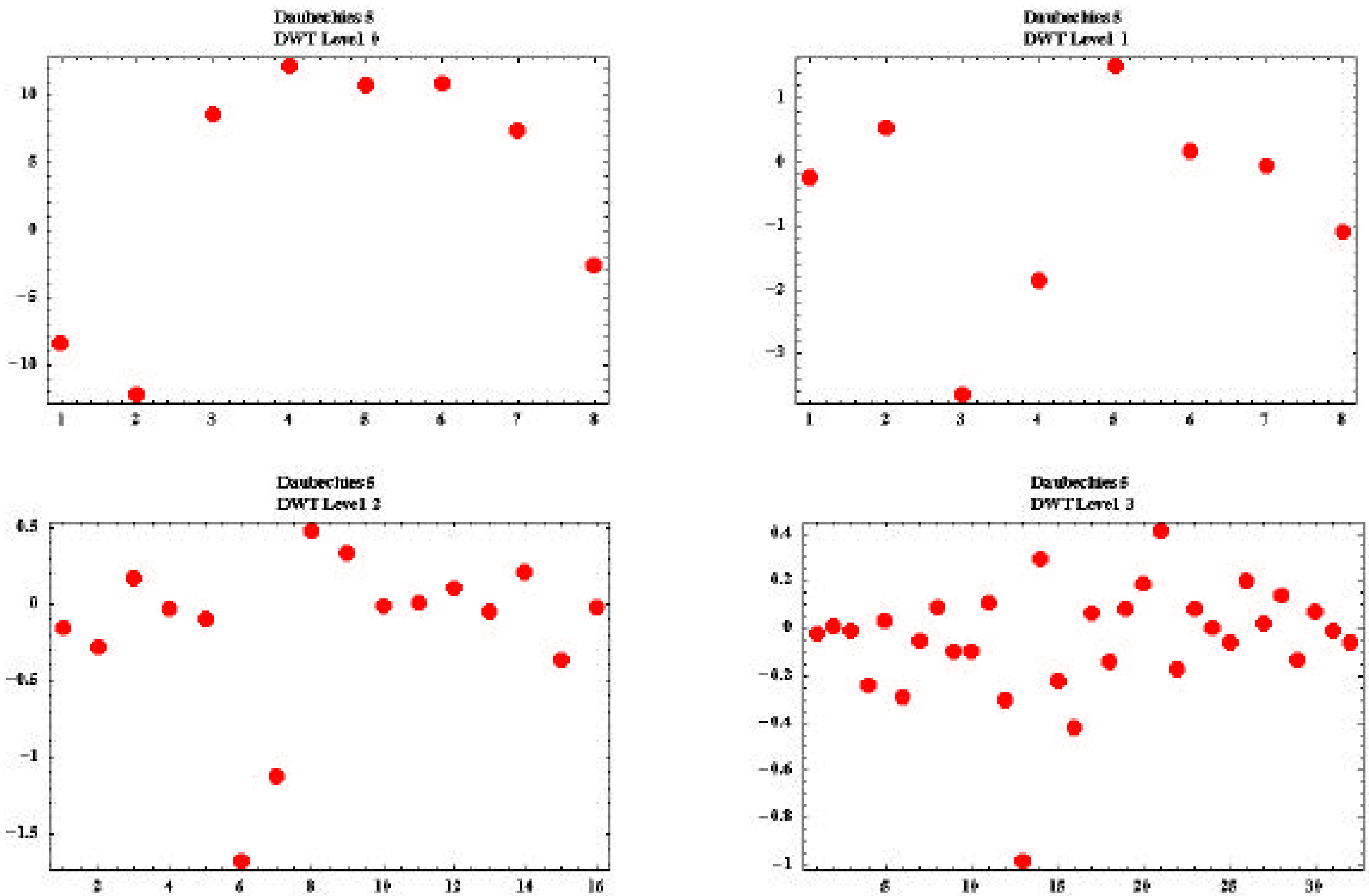
The First Four Levels of the MRD of Noisy Bolometer Energy Data Using Duabechies 5 Wavelets



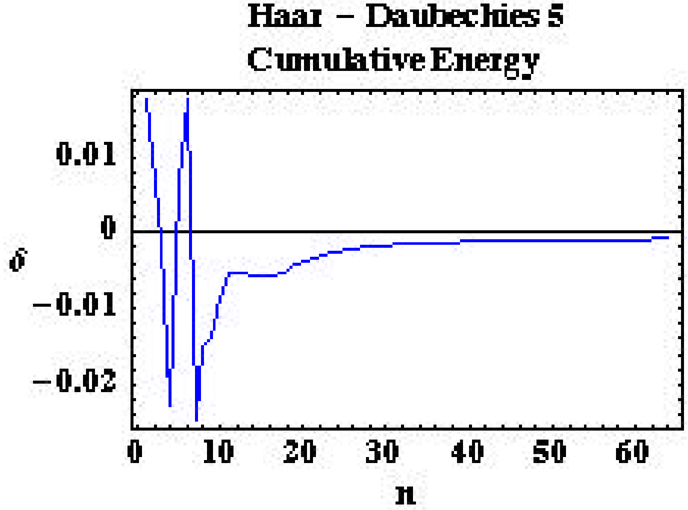
The Coefficients of the First 4 Levels of MRD Using D5 of Noisy Bolometer Energy Data

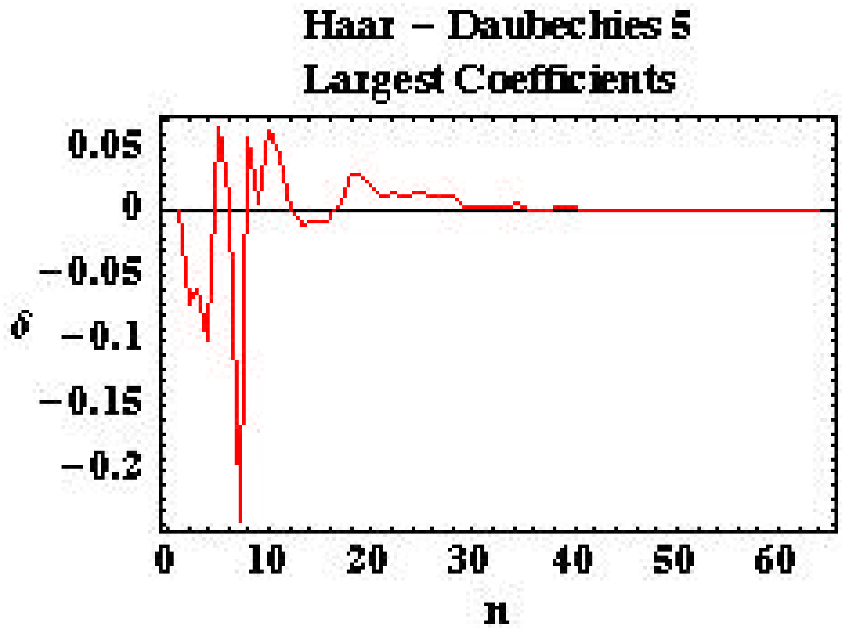
Polymath Research Inc.



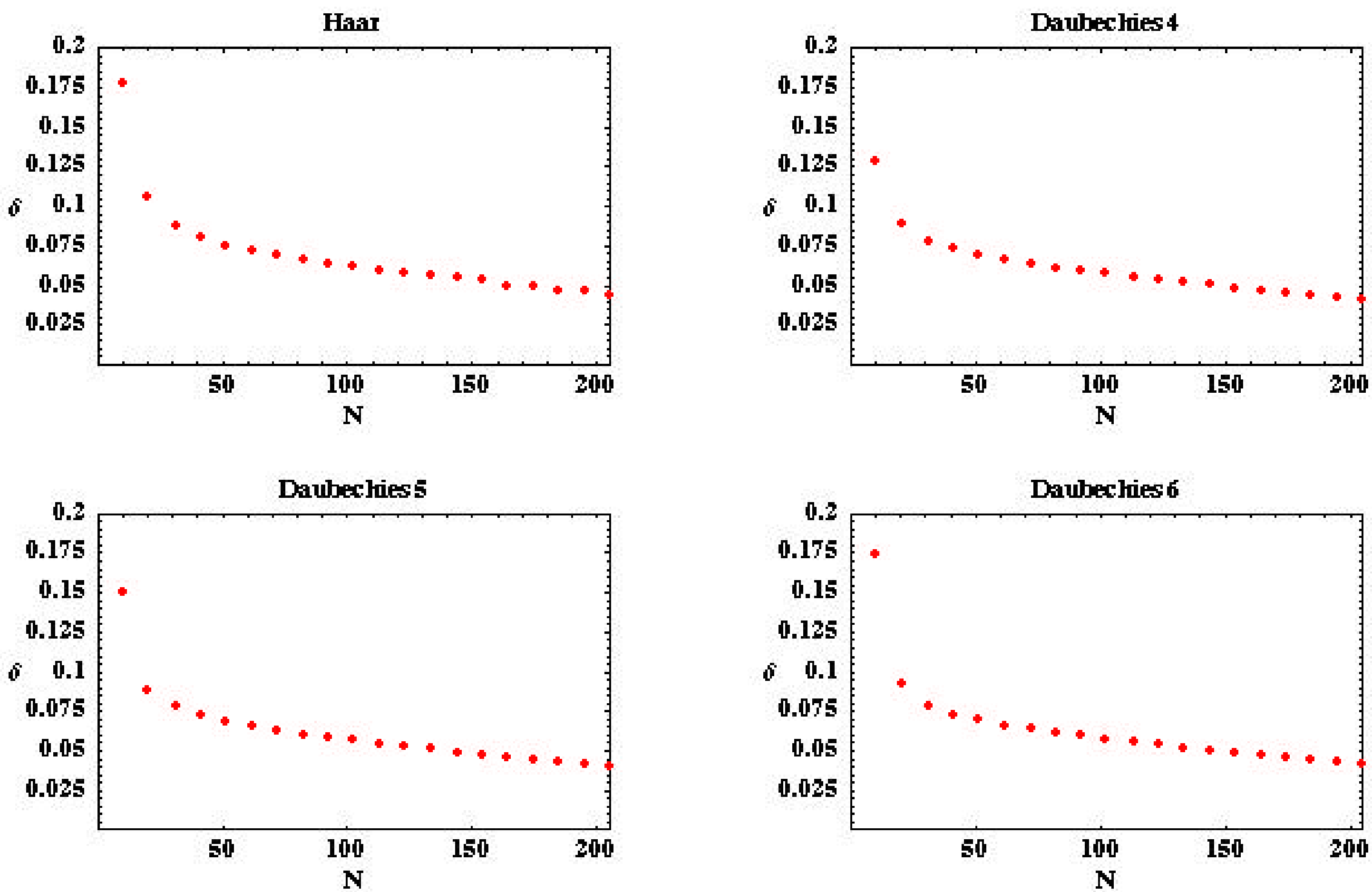
Daubchies 5 Does Better than Haar and Here's proof

Polymath Research Inc.

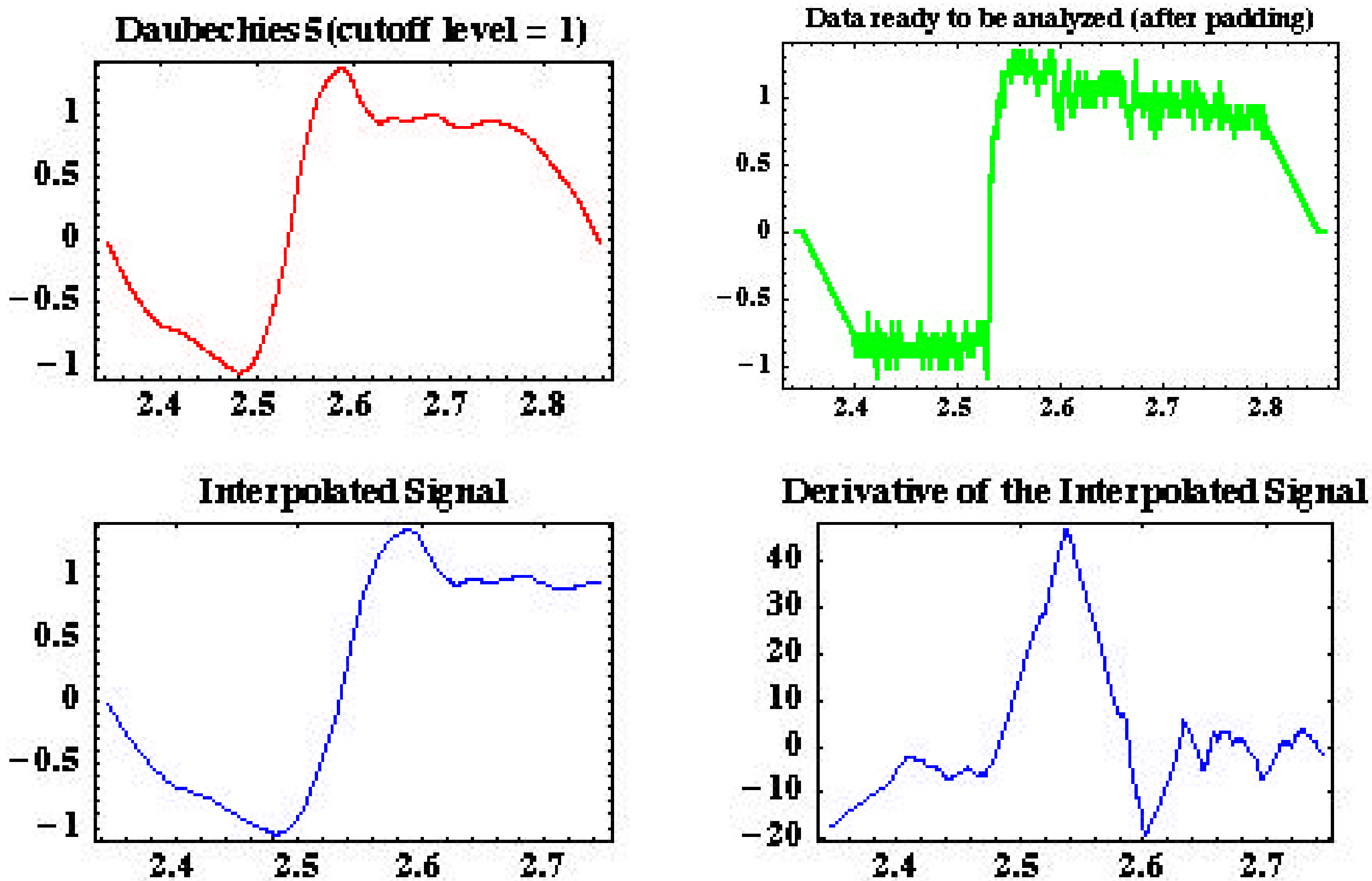




And Here's More Proof Why Daubechies 5 is Better Despite the Elevated Noise Levels...

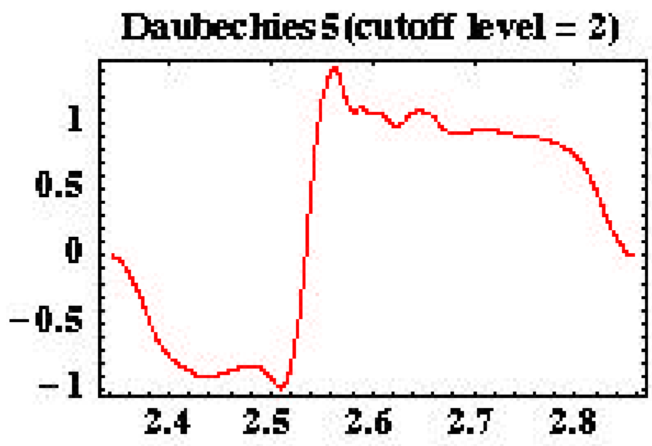


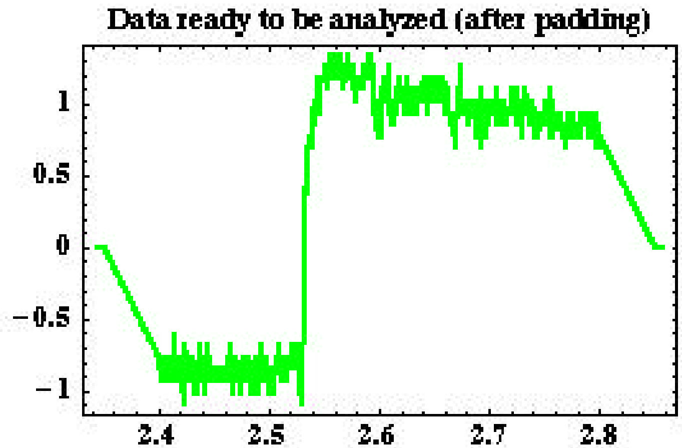
Partial Reconstruction of Noisy Bolometer Energy & Power with the Lowest MRD Level Using d5

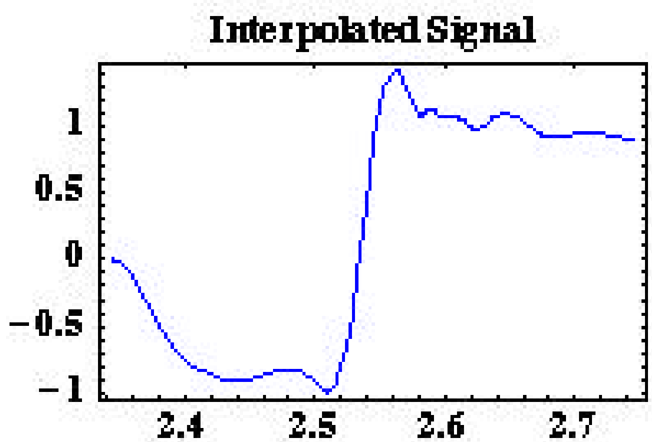


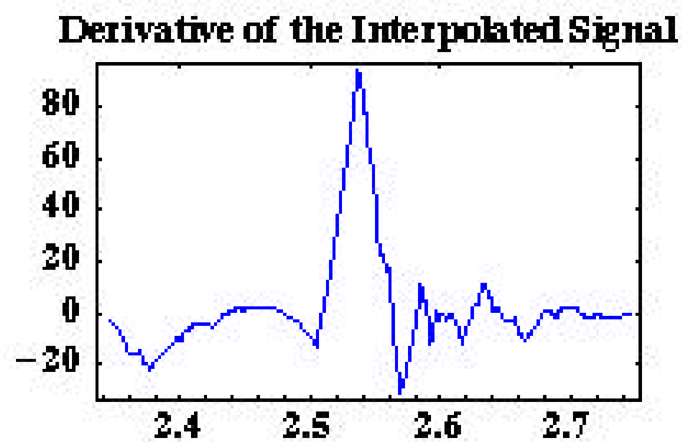
Partial Reconstruction of Noisy Bolometer Energy & Power with the 2 Lowest MRD Levels Using d5

Polymath Research Inc.



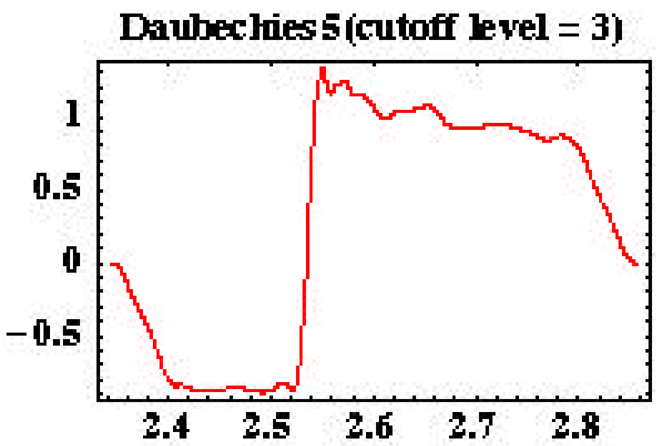


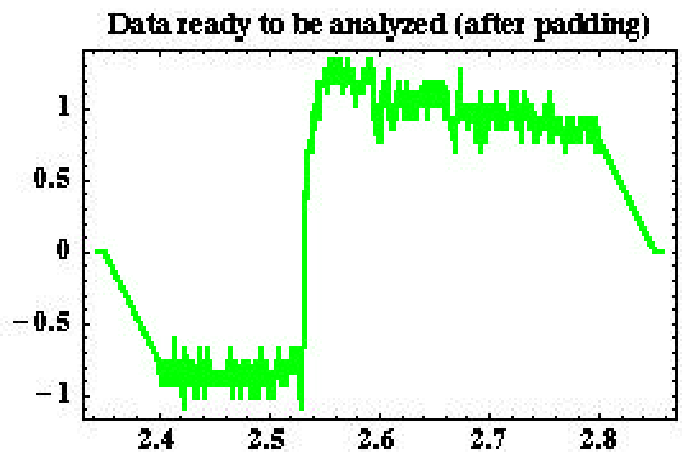


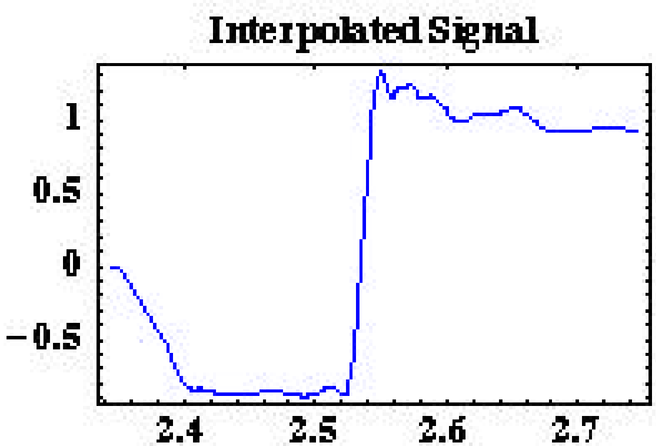


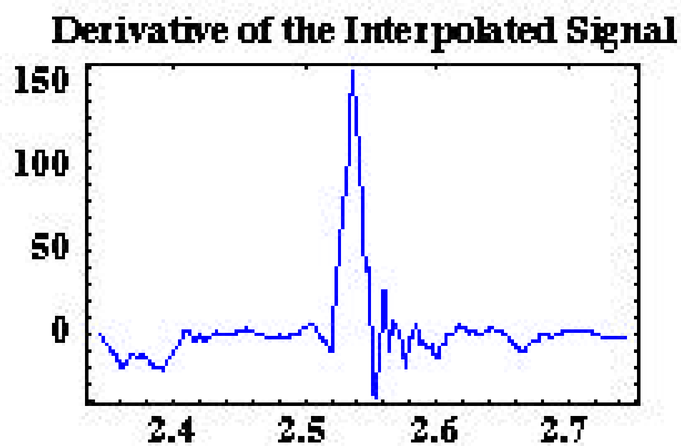
Partial Reconstruction of Noisy Bolometer Energy & Power with the 3 Lowest MRD Levels Using d5

Polymath Research Inc.



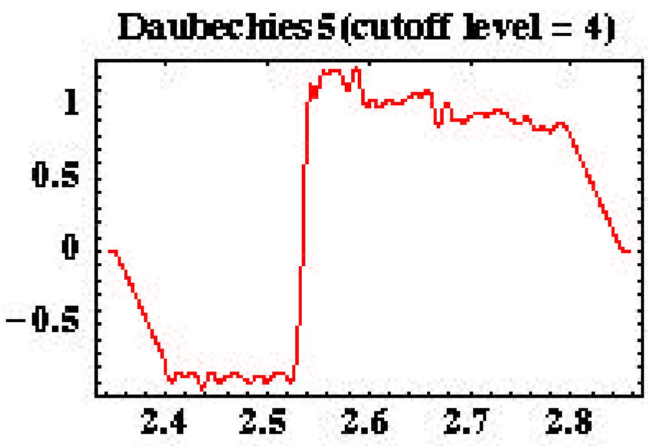


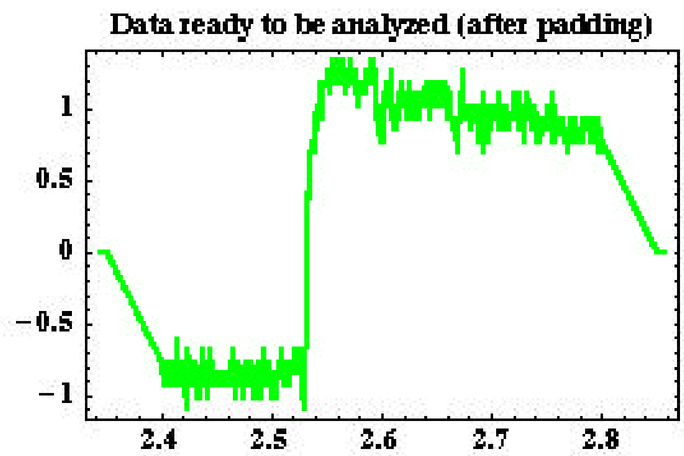


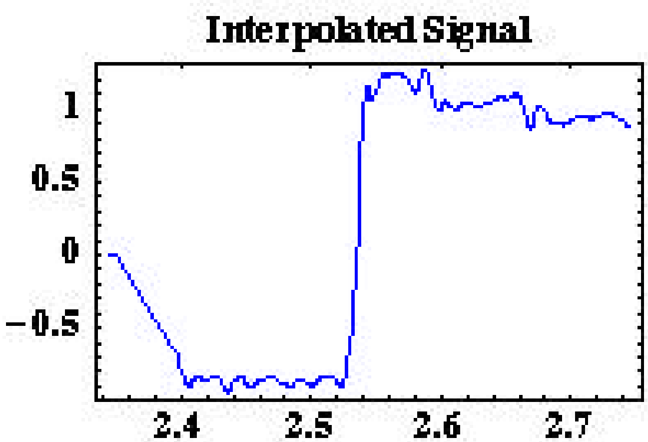


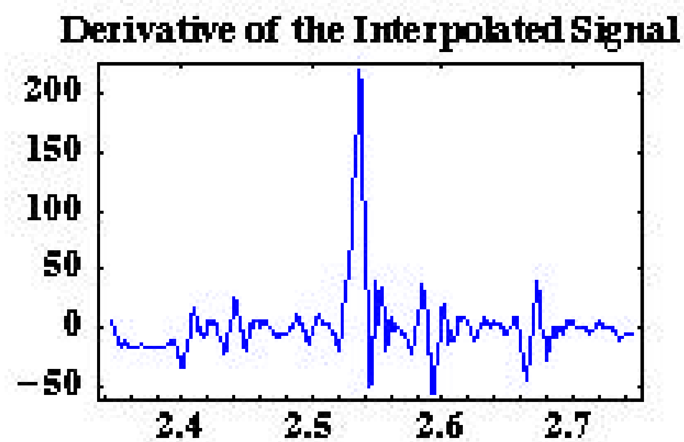
Partial Reconstruction of Noisy Bolometer Energy & Power with the 4 Lowest MRD Levels Using d5

Polymath Research Inc.

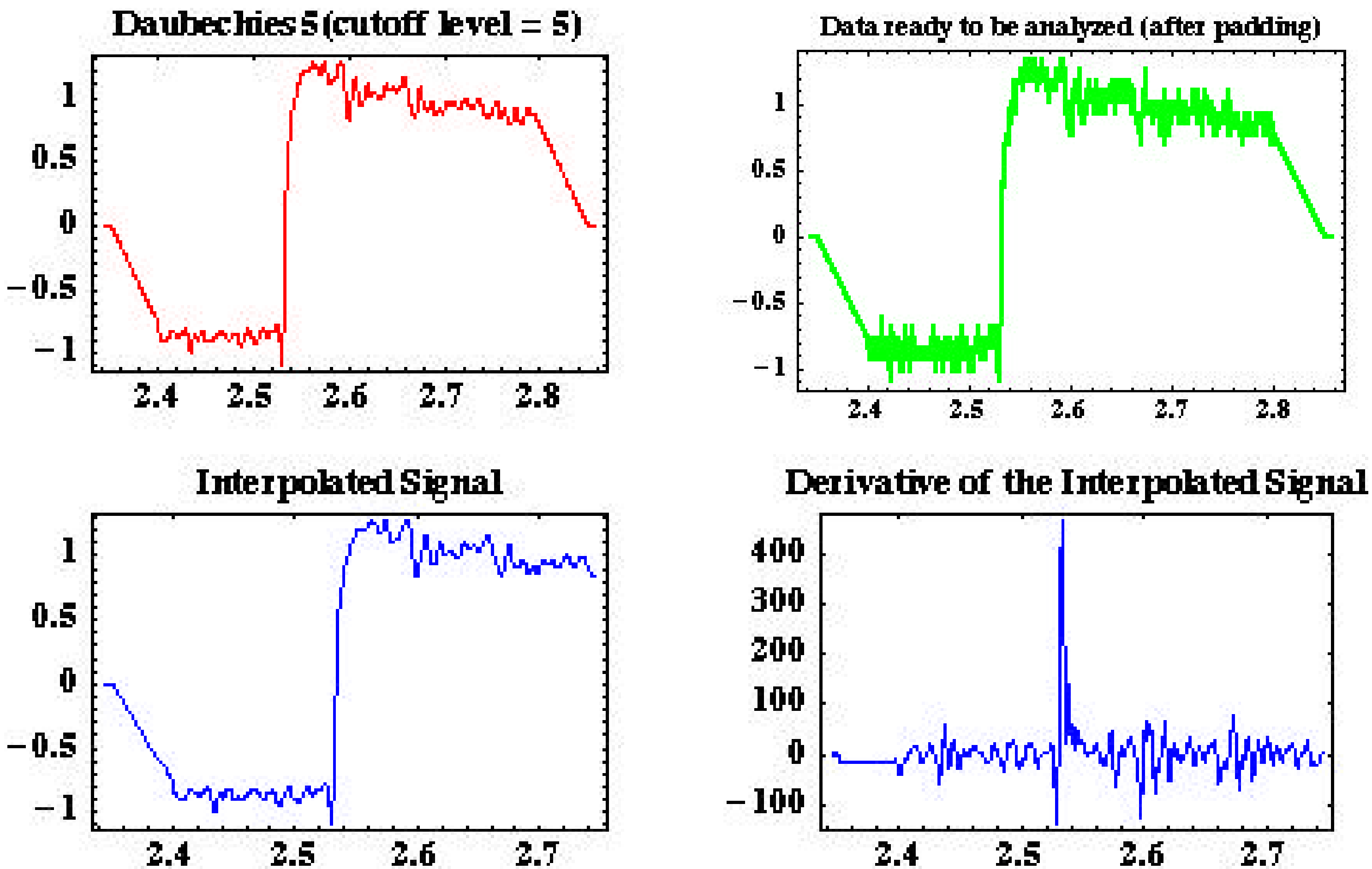






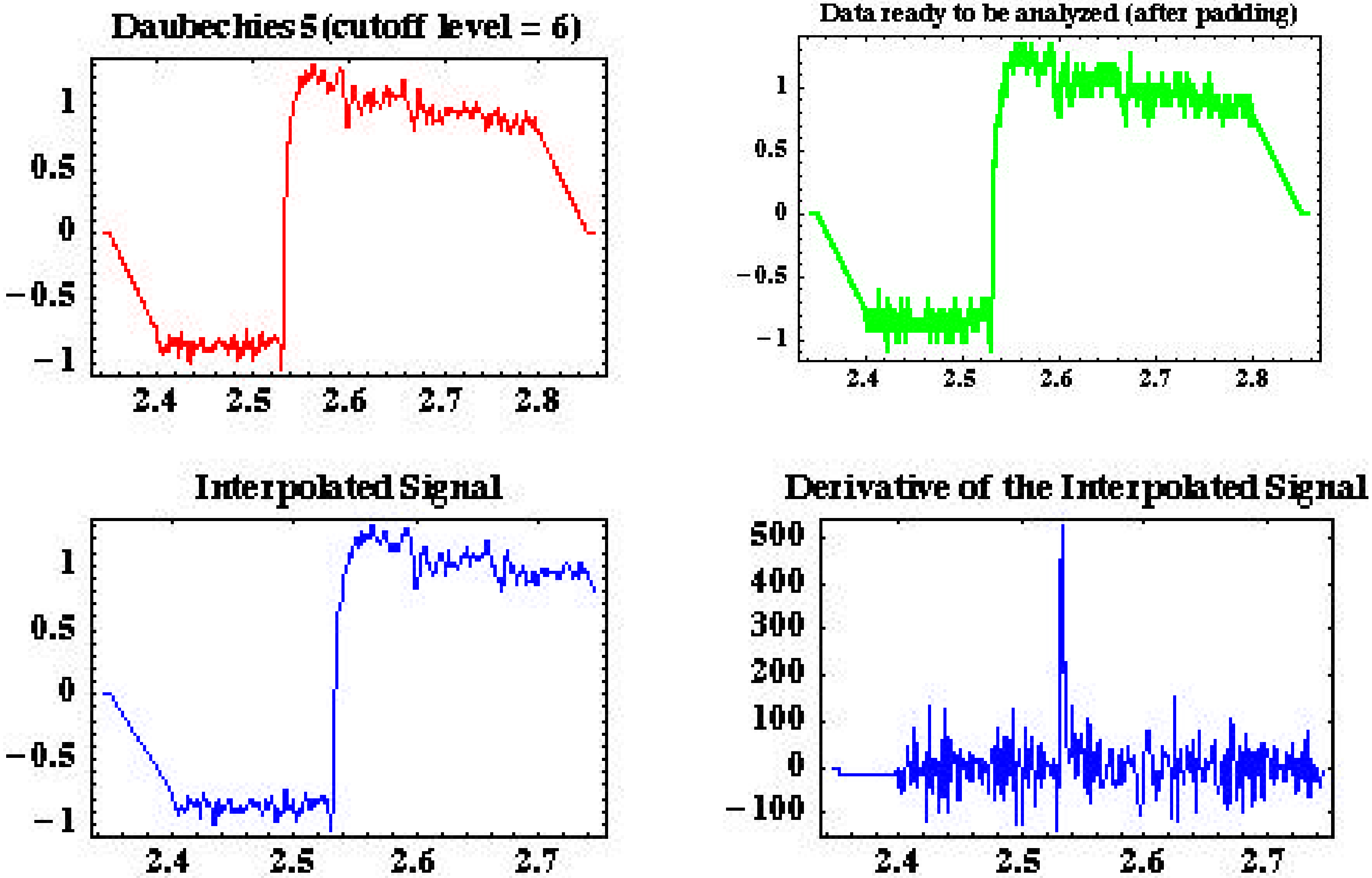


Partial Reconstruction of Noisy Bolometer Energy & Power with the 5 Lowest MRD Levels Using d5

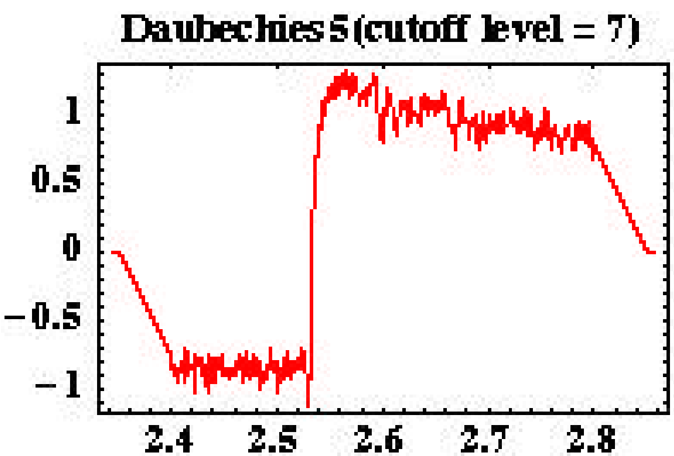


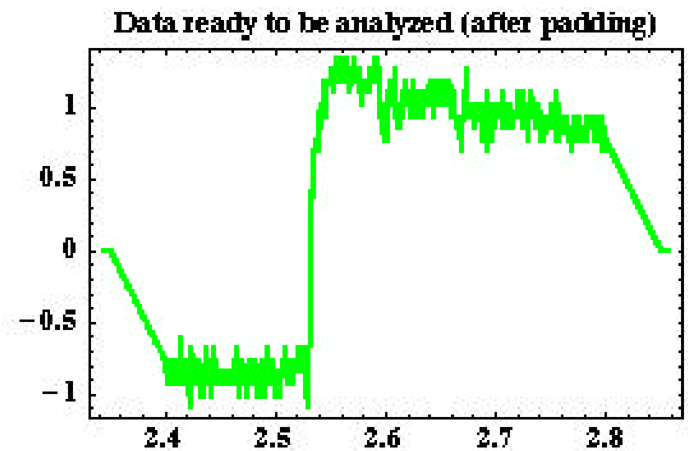
Partial Reconstruction of Noisy Bolometer Energy & Power with the 6 Lowest MRD Levels Using d5

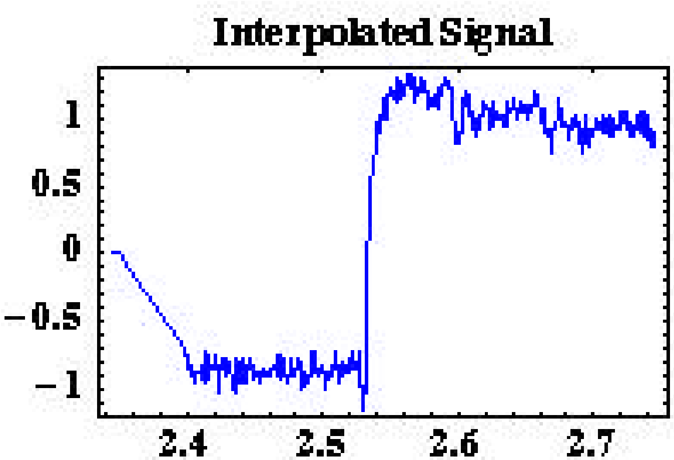
Polymath Research Inc.

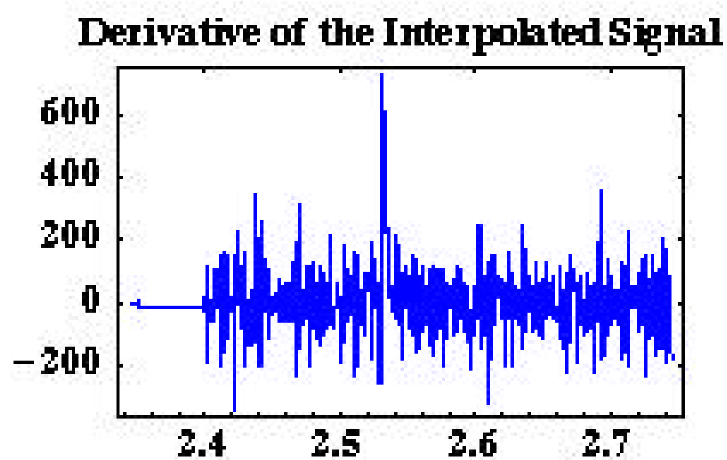


Partial Reconstruction of Noisy Bolometer Energy & Power with the 7 Lowest MRD Levels Using d5

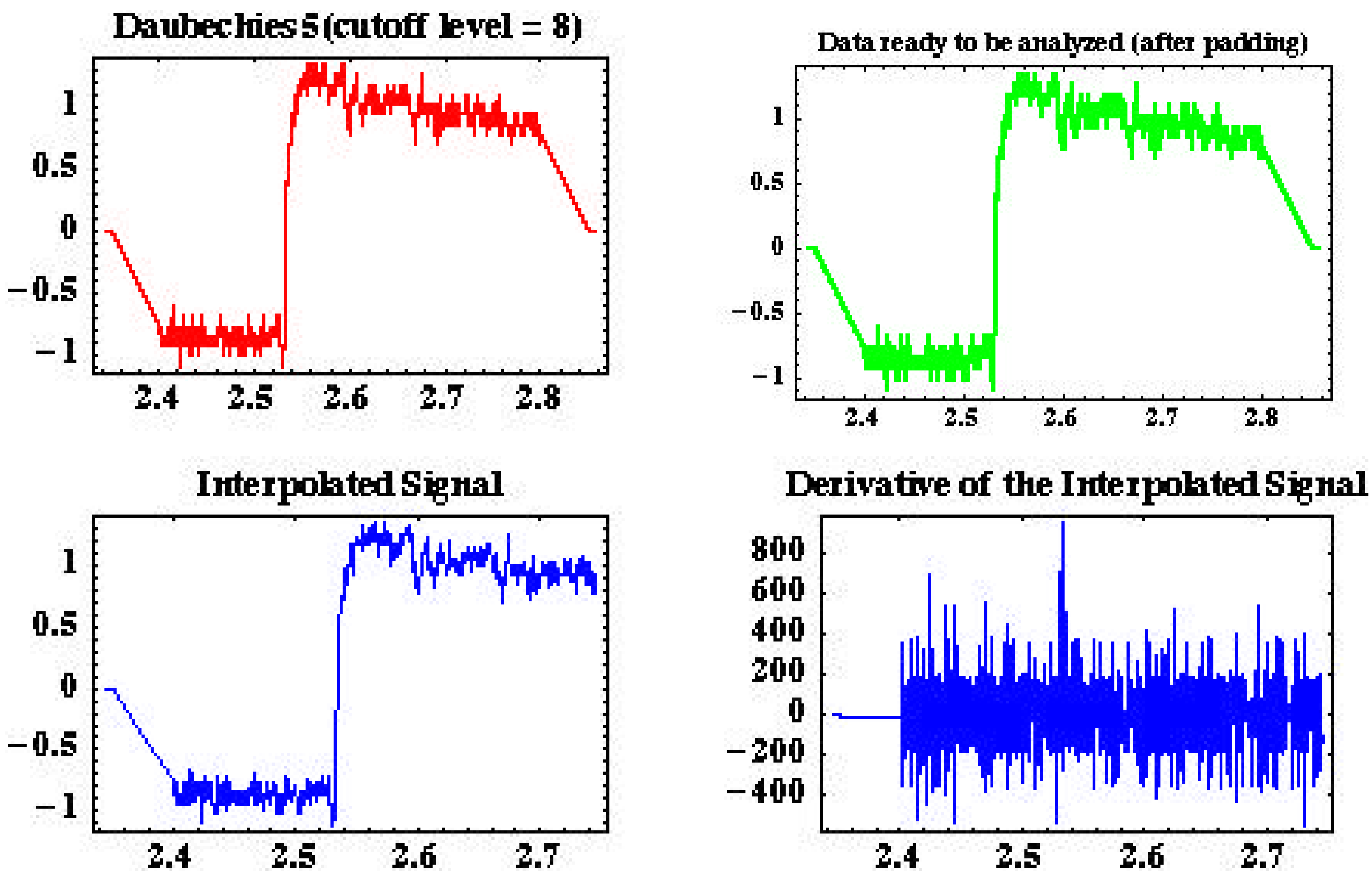








Partial Reconstruction of Noisy Bolometer Energy & Power with the 8 Lowest MRD Levels Using d5



Reconstruction of Noisy Bolometer Energy & Power with All 9 MRD Levels Using d5

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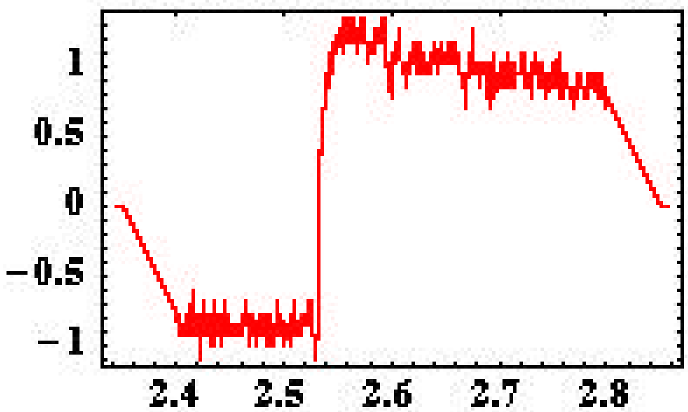
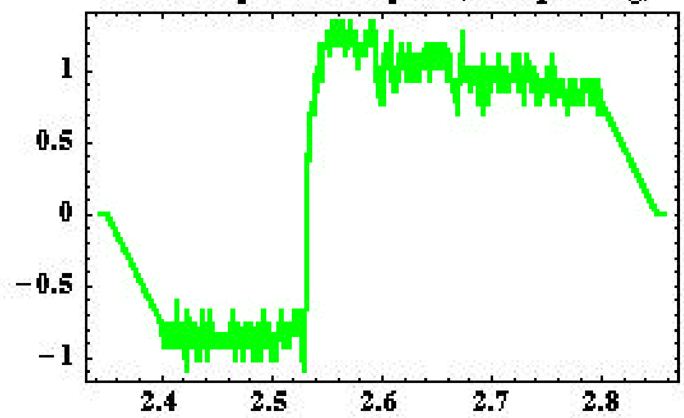
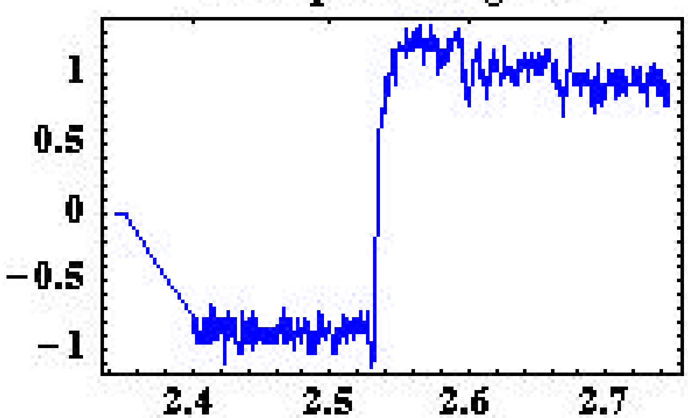
Daubechies 5 (cutoff level = 8)



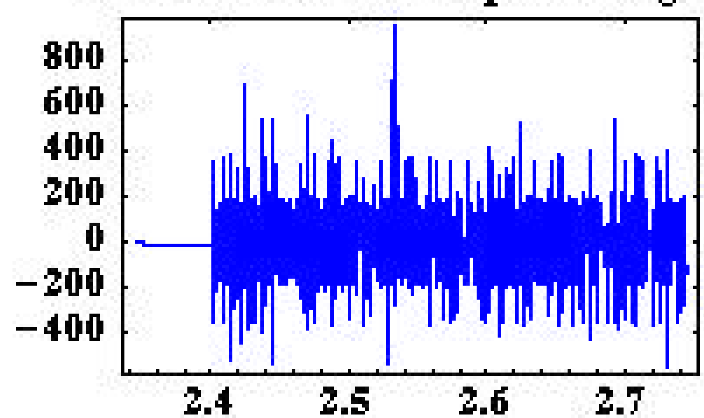
Data ready to be analyzed (after padding)



Interpolated Signal

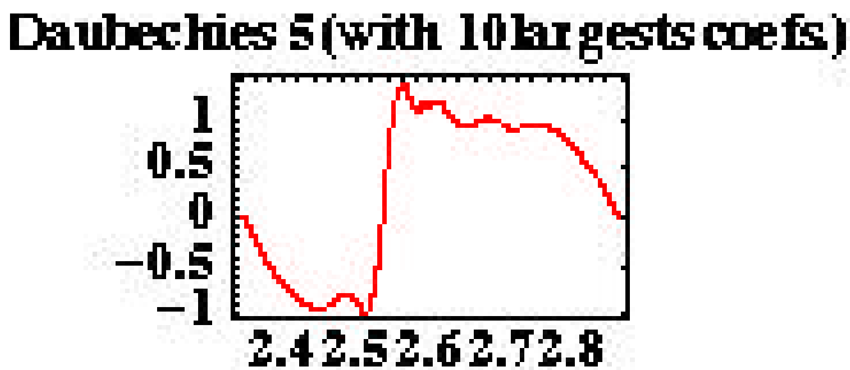


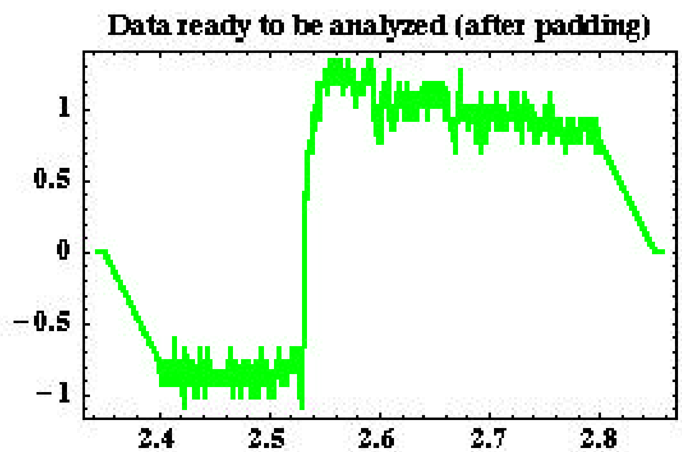
Derivative of the Interpolated Signal

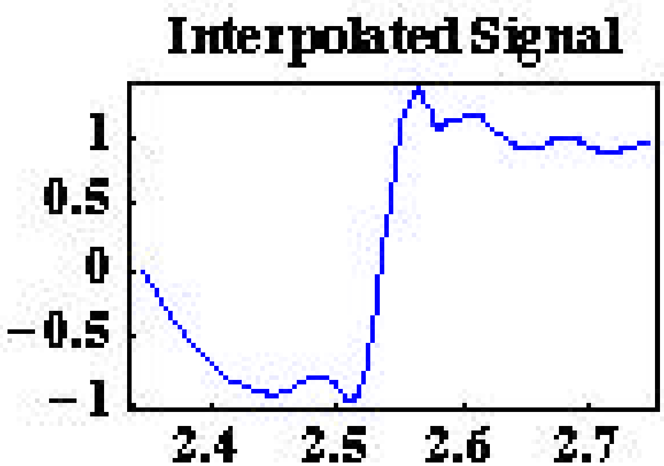


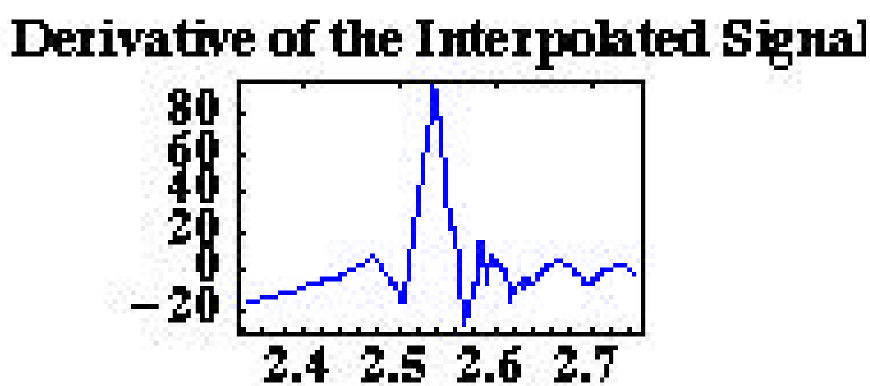
Partial Reconstruction of the Energy & Power with 10 Largest Coefficients Kept Using d5

Polymath Research Inc.

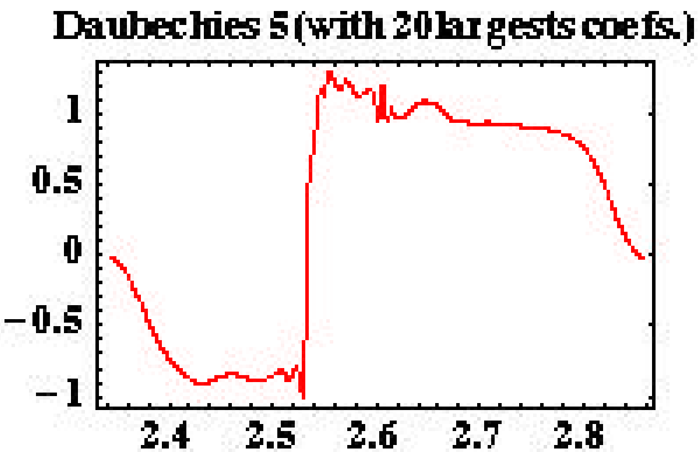


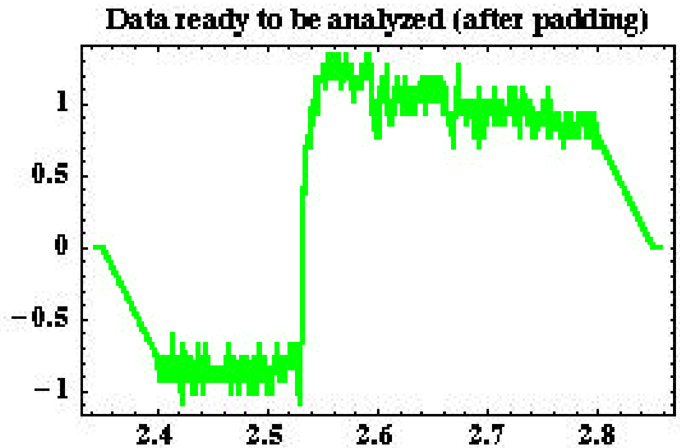


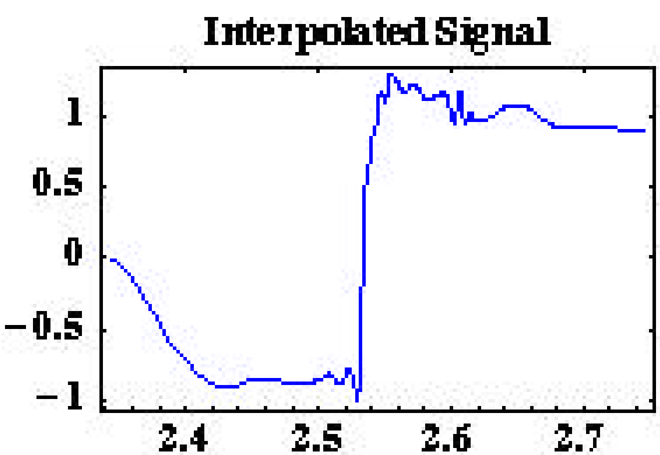


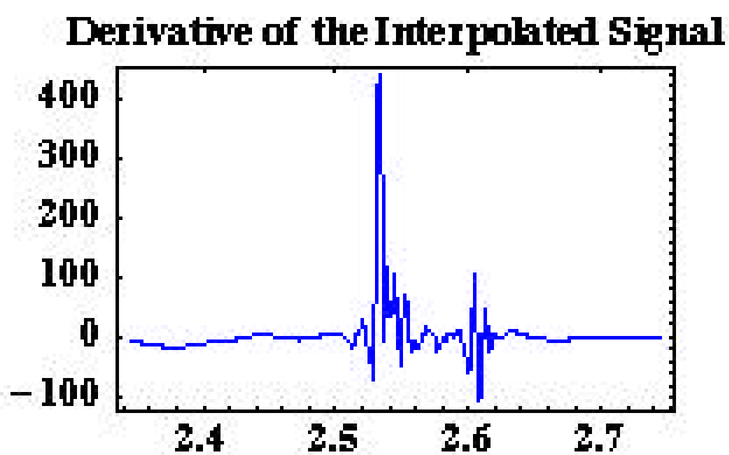


Partial Reconstruction of the Energy & Power with 20 Largest Coefficients Kept Using d5



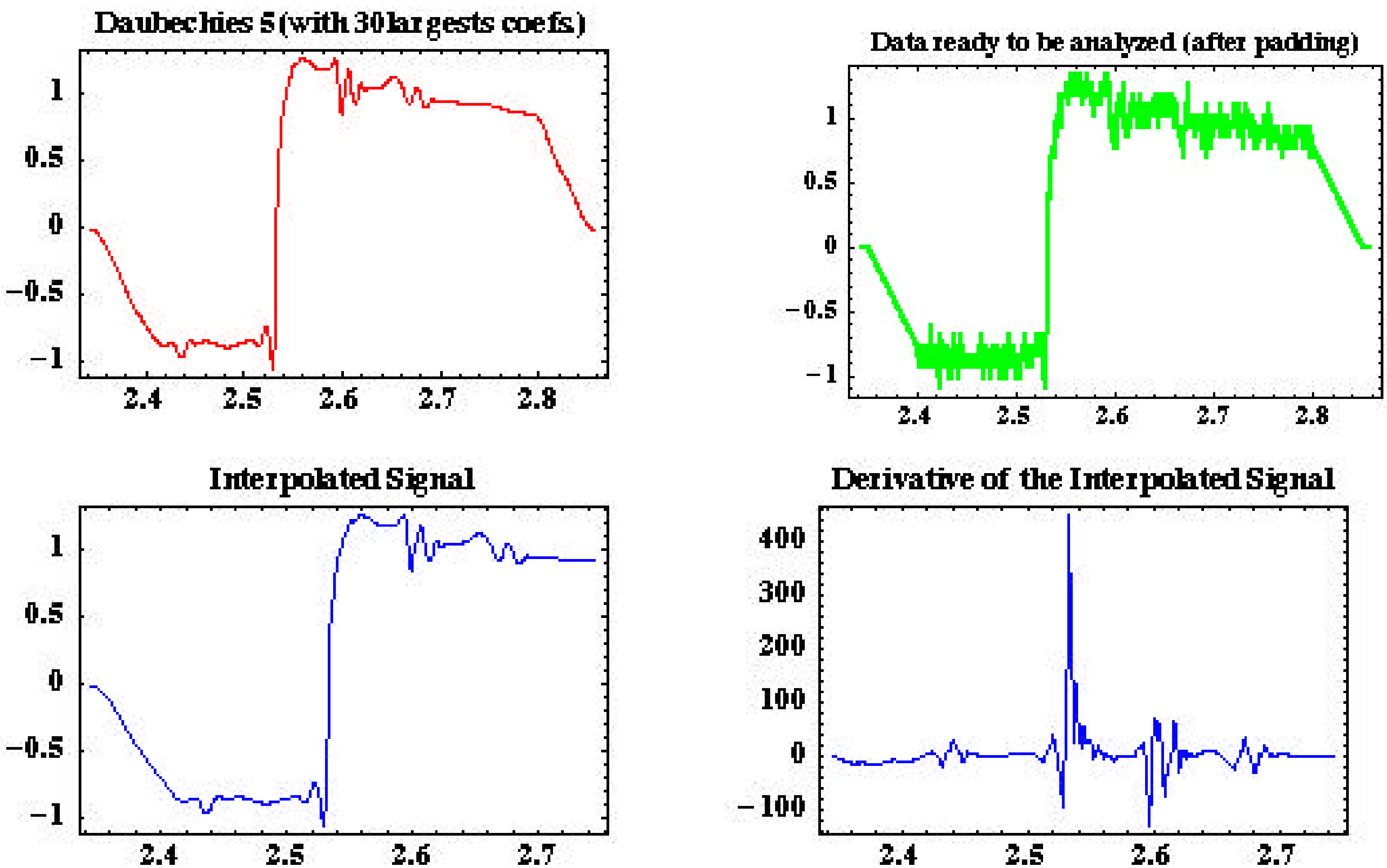




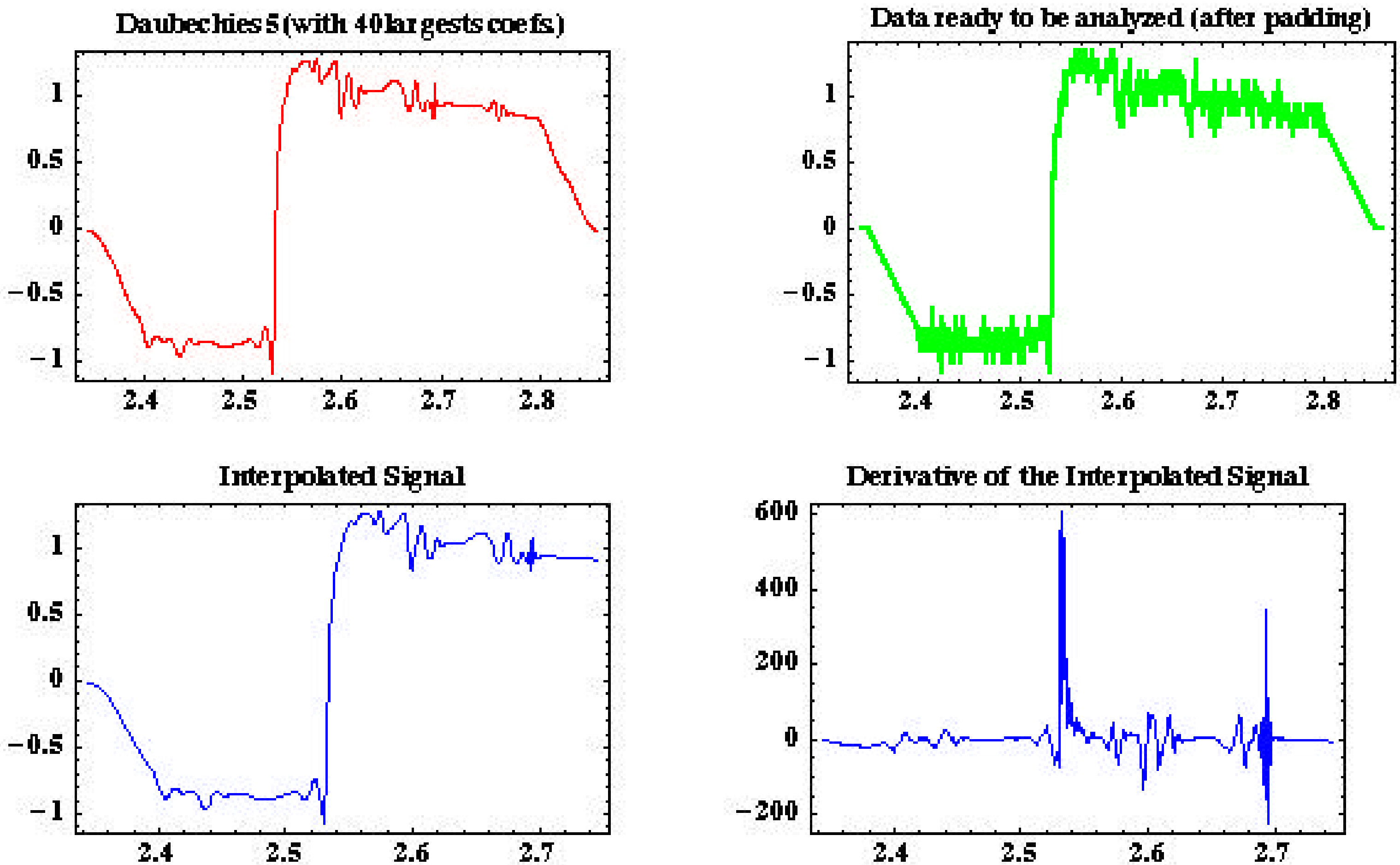


Partial Reconstruction of the Energy & Power with 30 Largest Coefficients Kept Using d5

Polymath Research Inc.

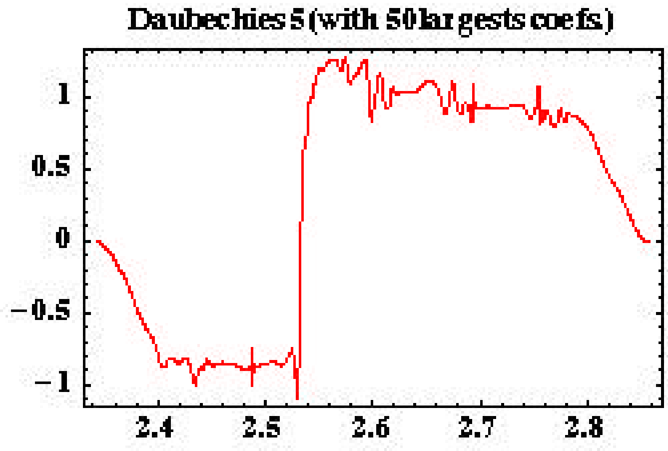


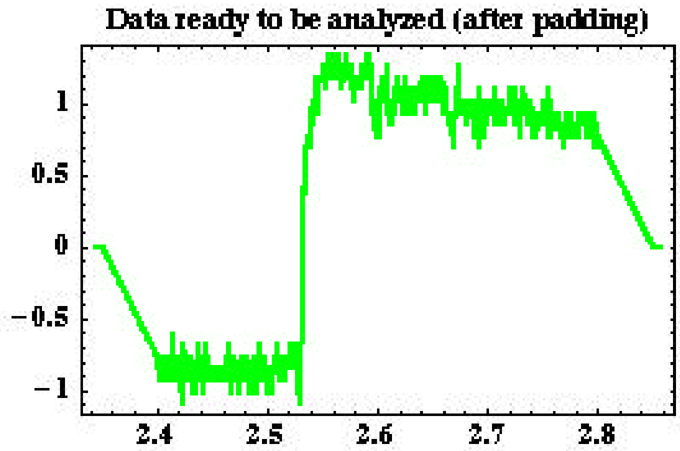
Partial Reconstruction of the Energy & Power with 40 Largest Coefficients Kept Using d5

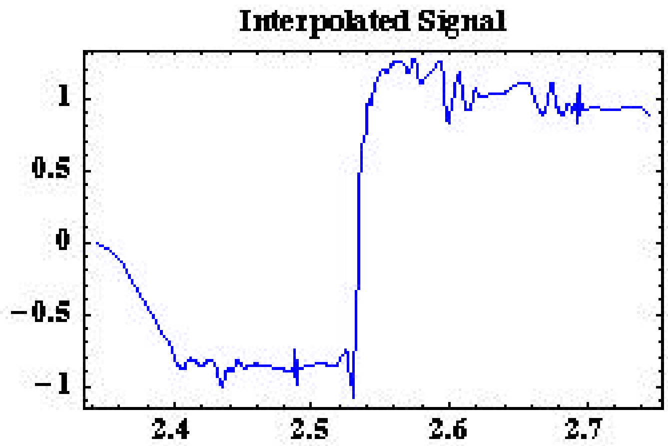


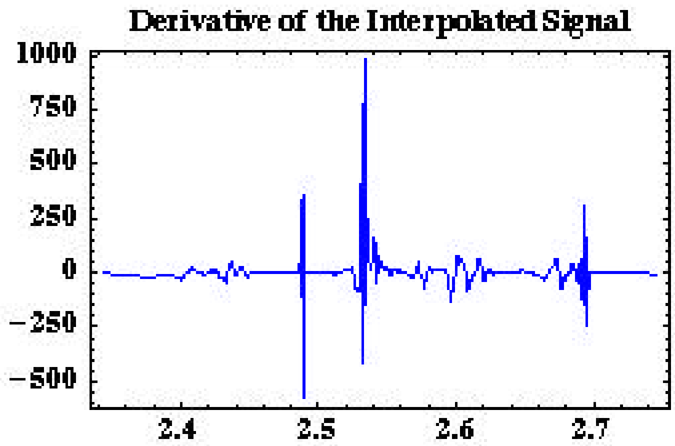
Partial Reconstruction of the Energy & Power with 50 Largest Coefficients Kept Using d5

Polymath Research Inc.

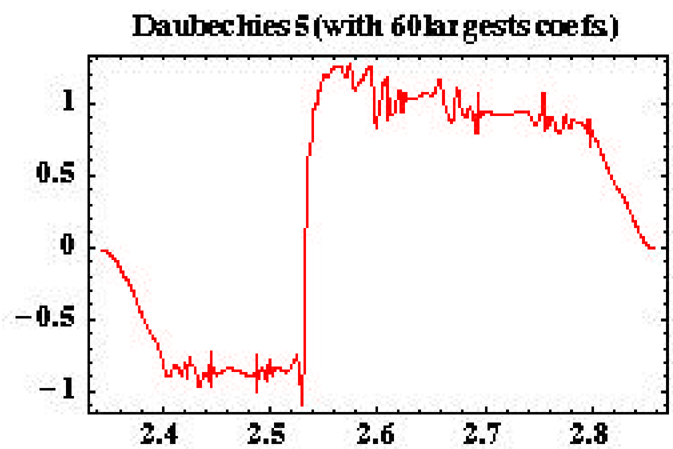


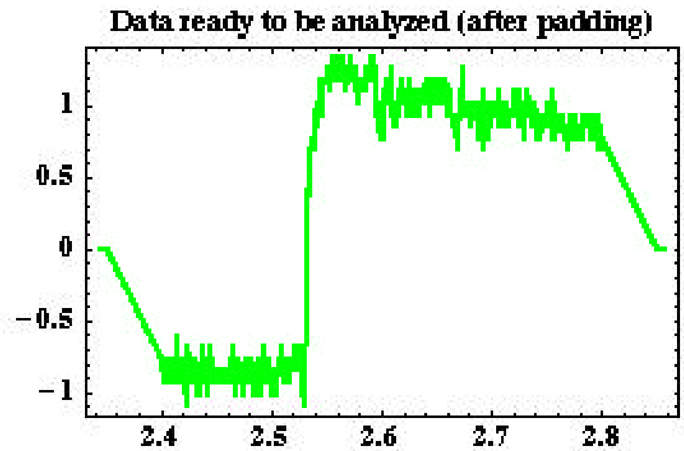


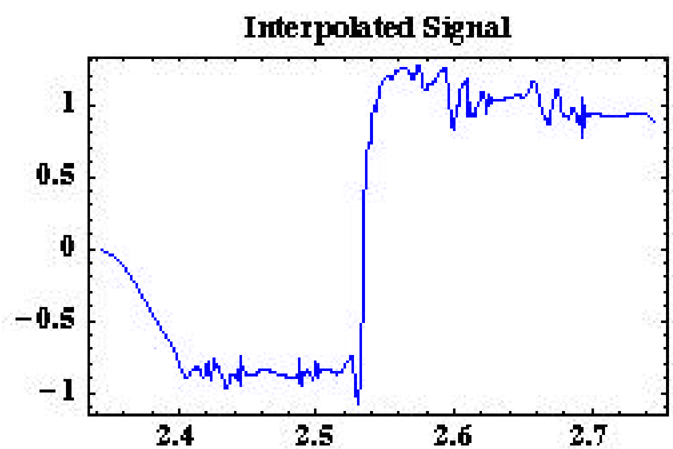


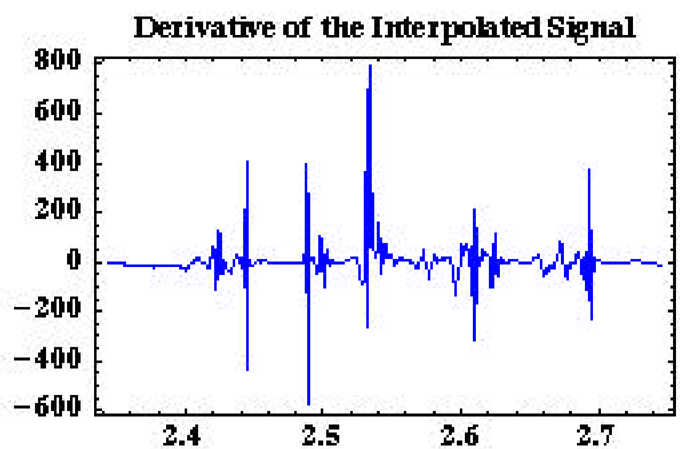


Partial Reconstruction of the Energy & Power with 60 Largest Coefficients Kept Using d5

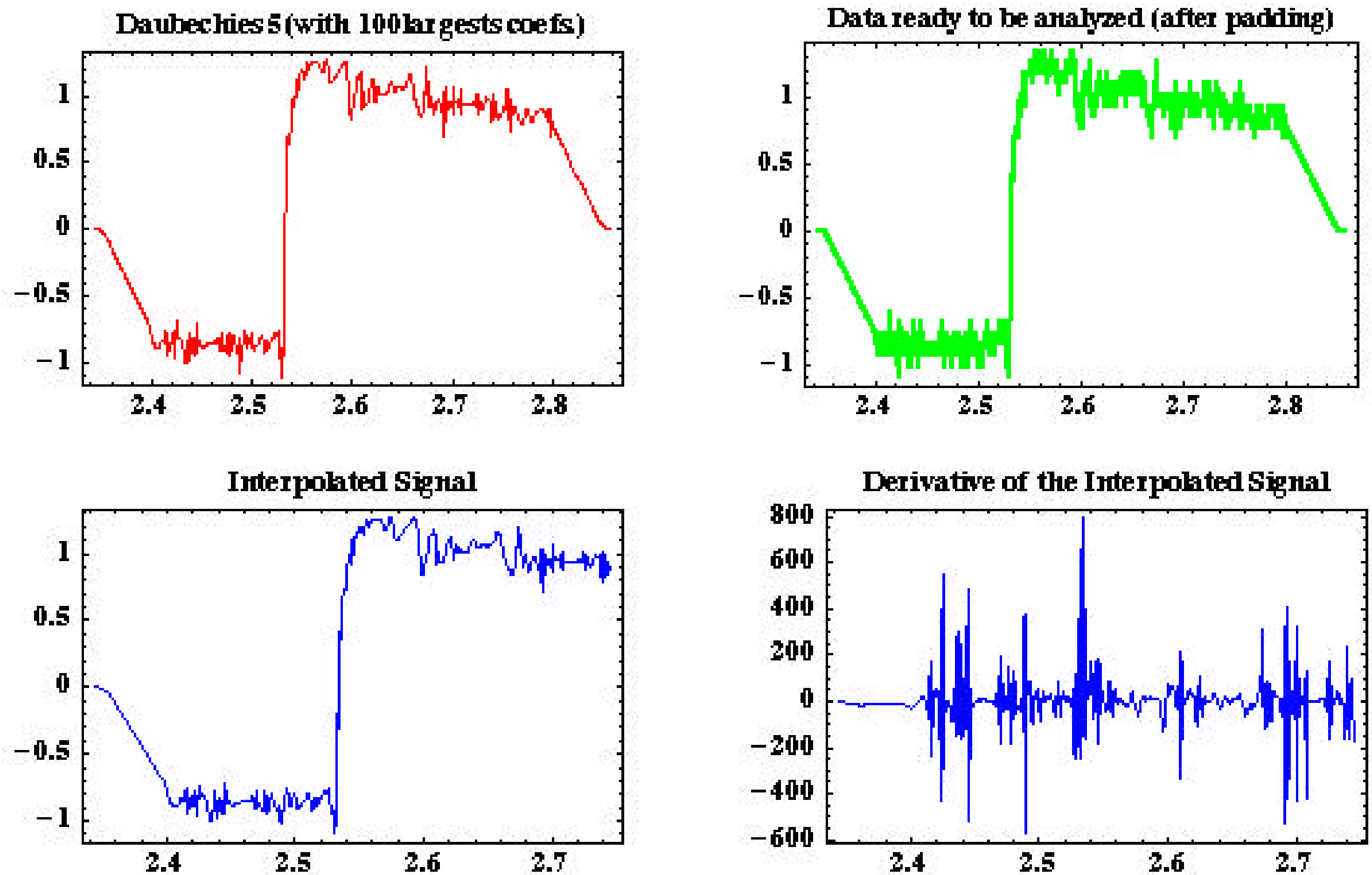




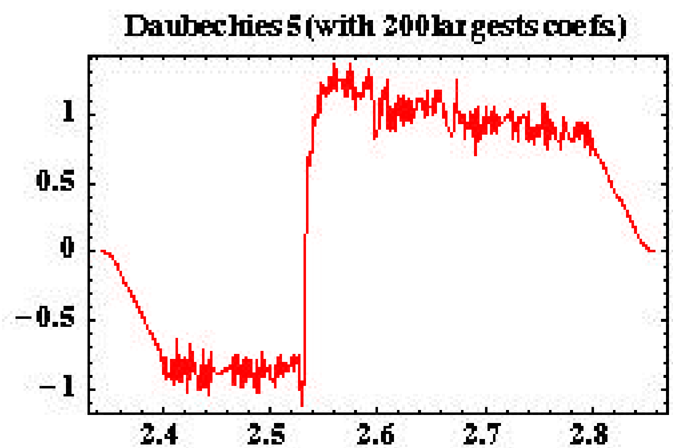


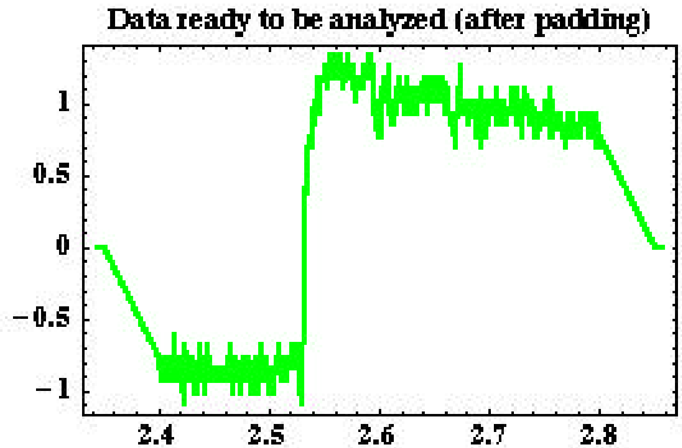


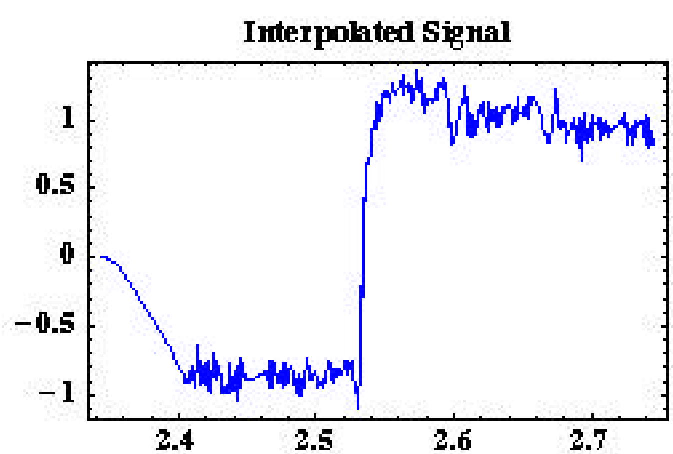
Partial Reconstruction of the Energy & Power with 100 Largest Coefficients Kept Using d5

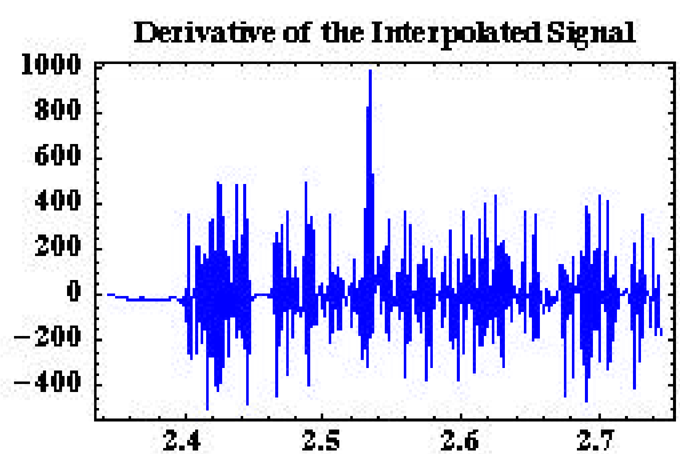


Partial Reconstruction of the Energy & Power with 200 Largest Coefficients Kept Using d5



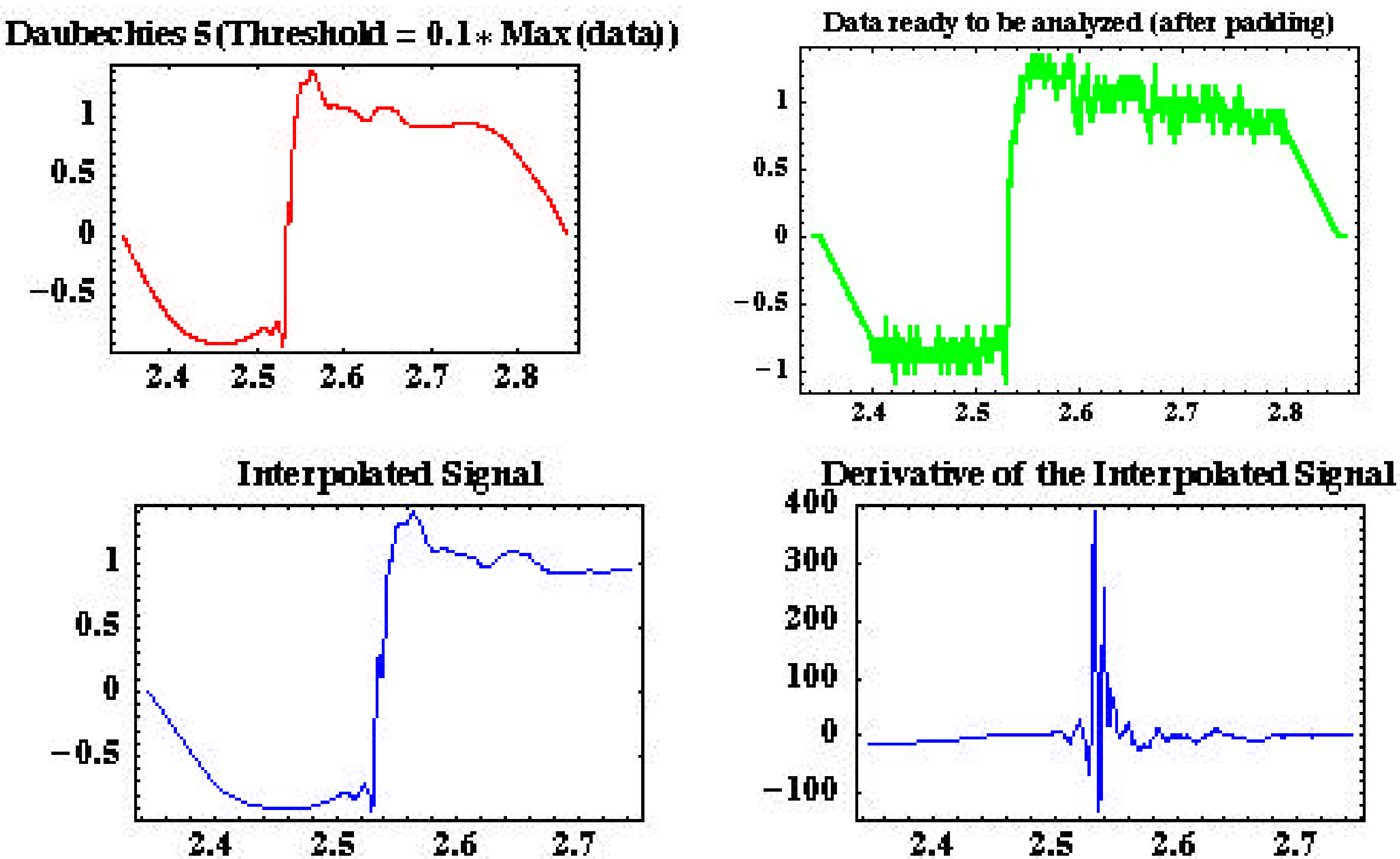






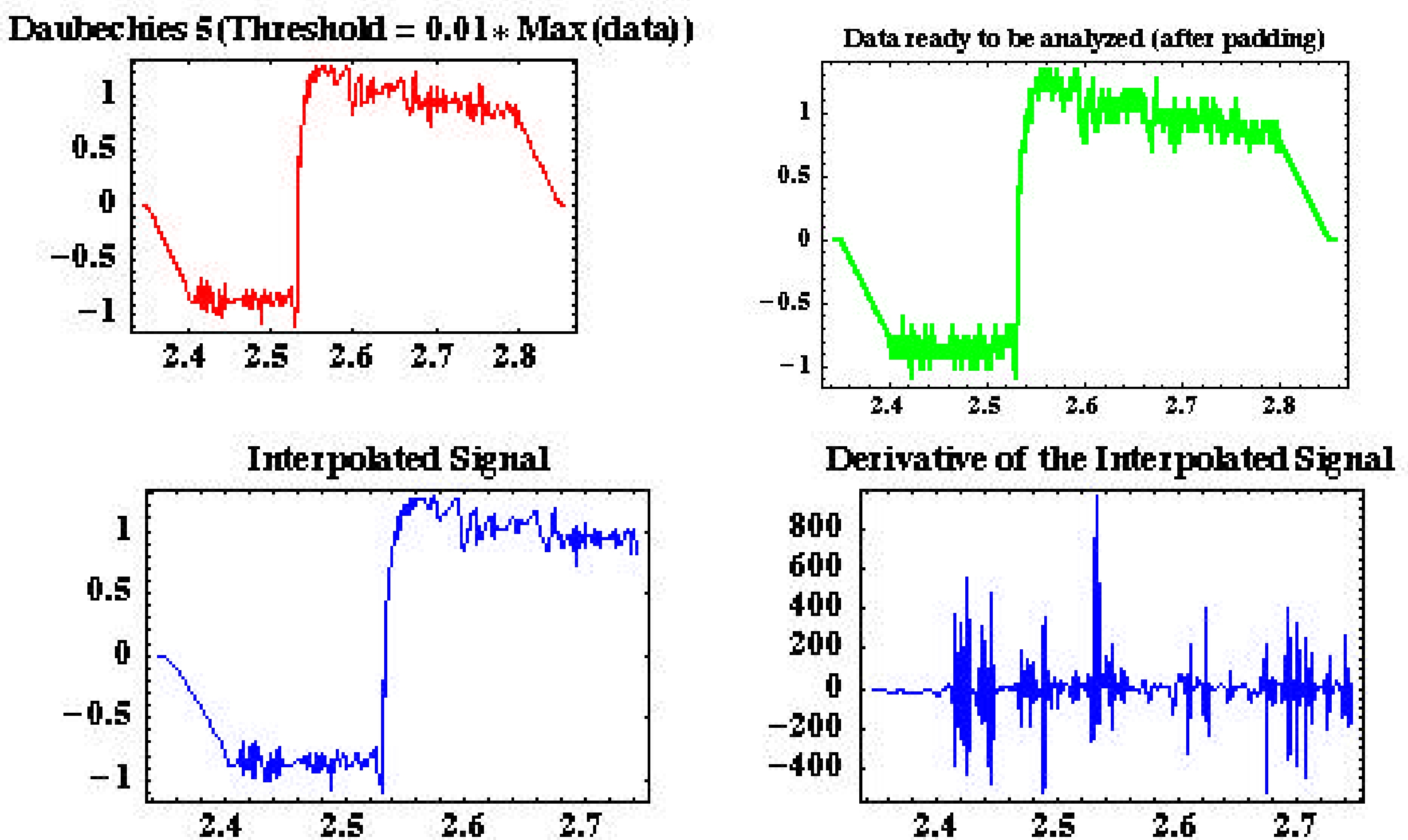
Reconstruction of Noisy Bolometer Energy & Power Keeping Up to 10% of Maximum Amplitude

Polymath Research Inc.



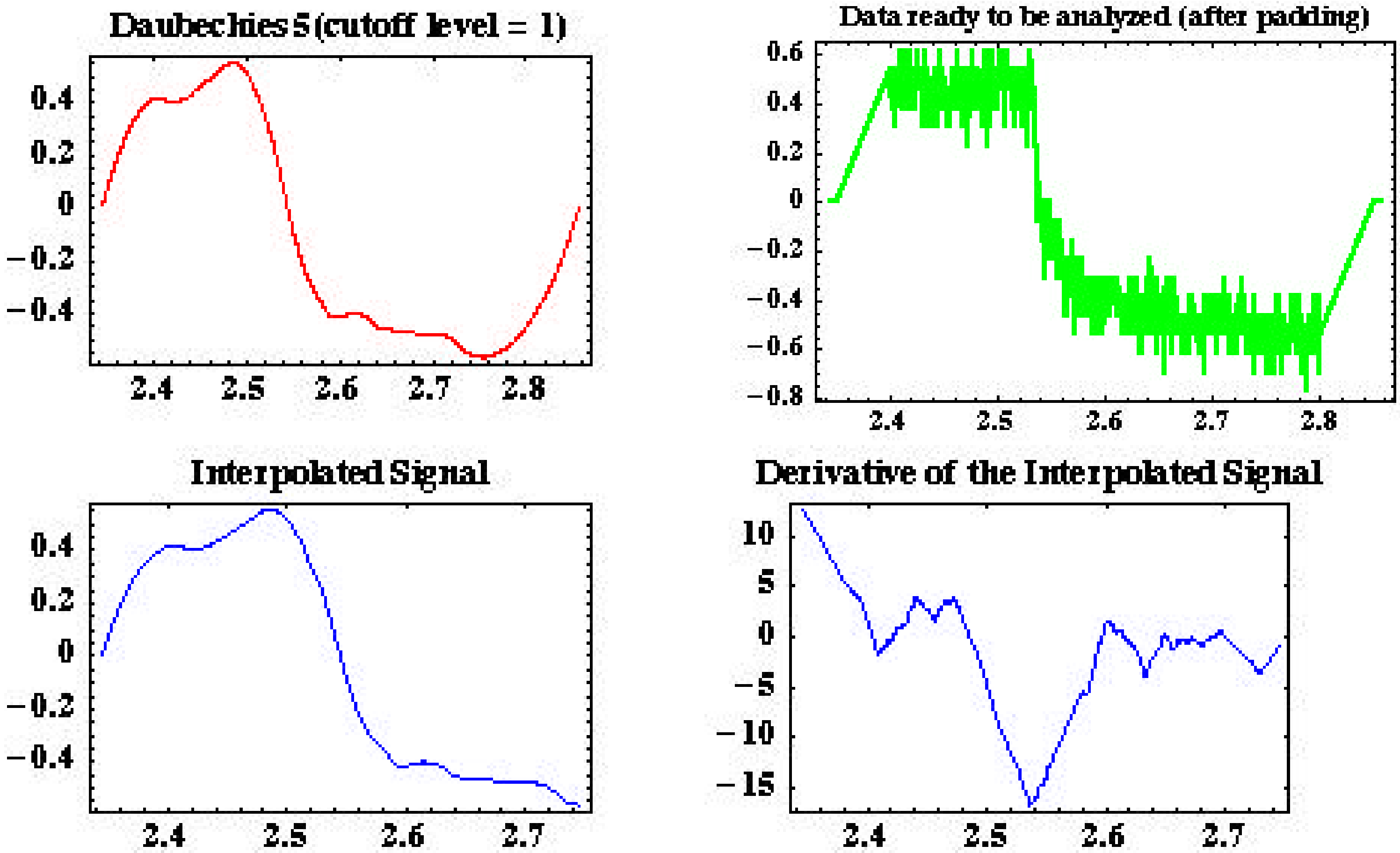
Reconstruction of Noisy Bolometer Energy & Power Keeping Up to 1% of Maximum Amplitude

Polymath Research Inc.

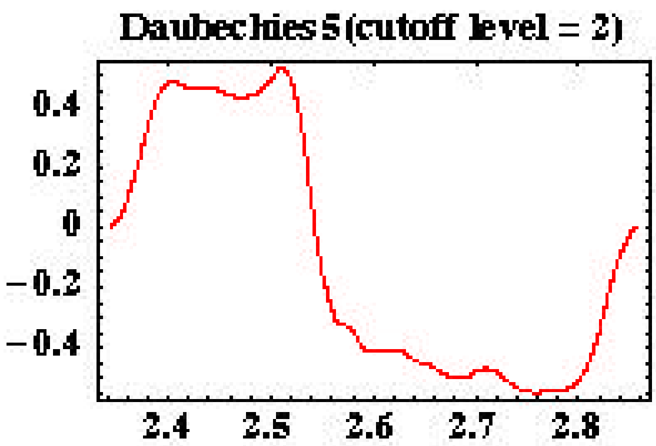


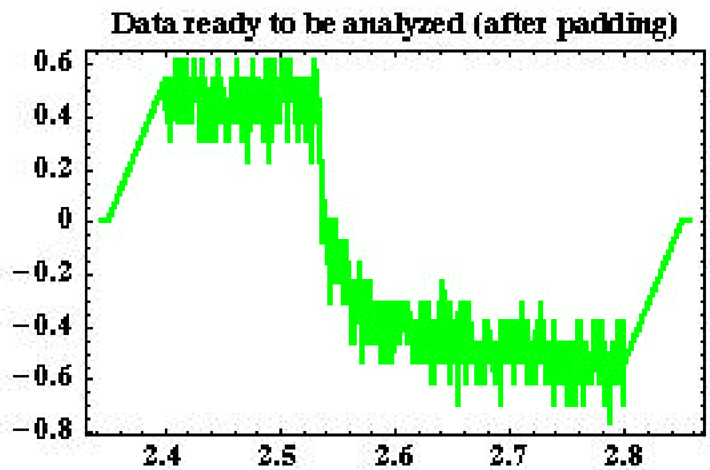
Reconstruction of Very Noisy Bolometer Energy & Power with Lowest MRD Level Using d5

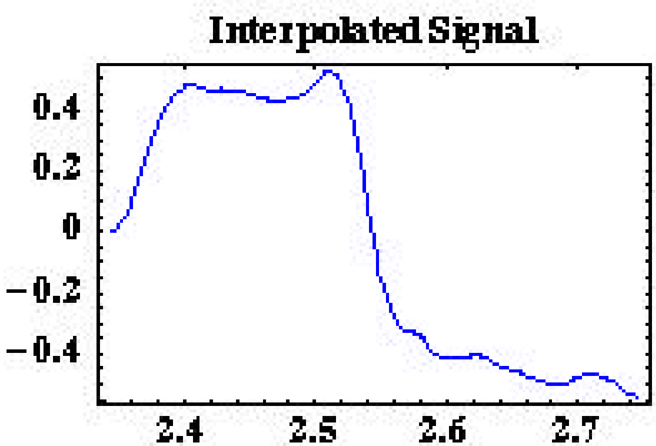
Polymath Research Inc.

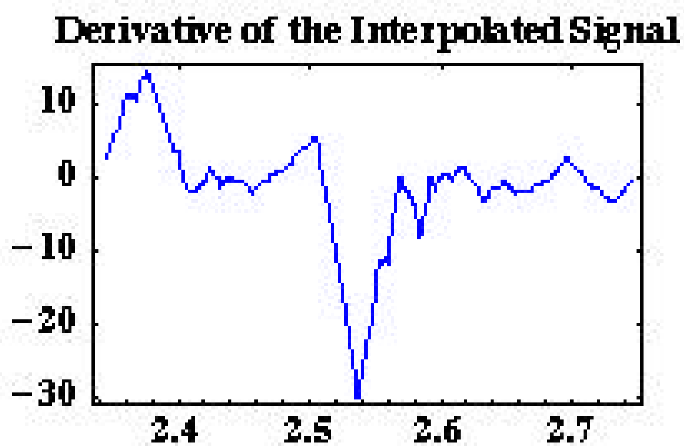


Reconstruction of Very Noisy Bolometer Energy & Power with 2 Lowest MRD Levels Using d5

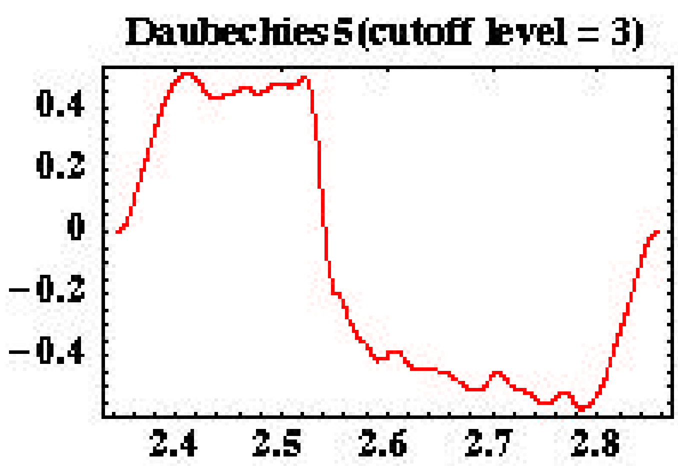


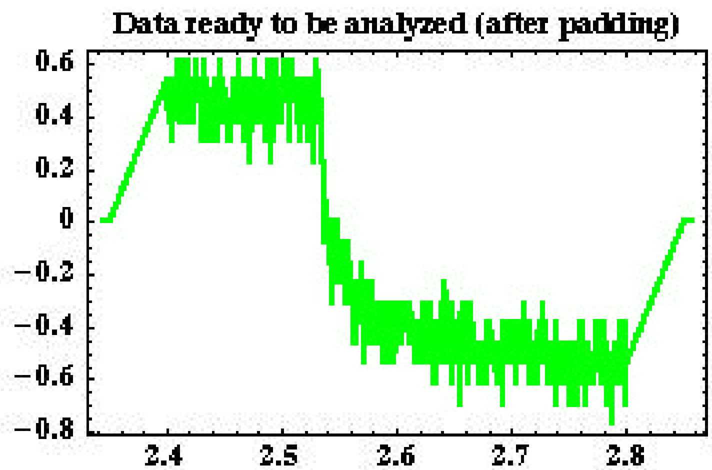


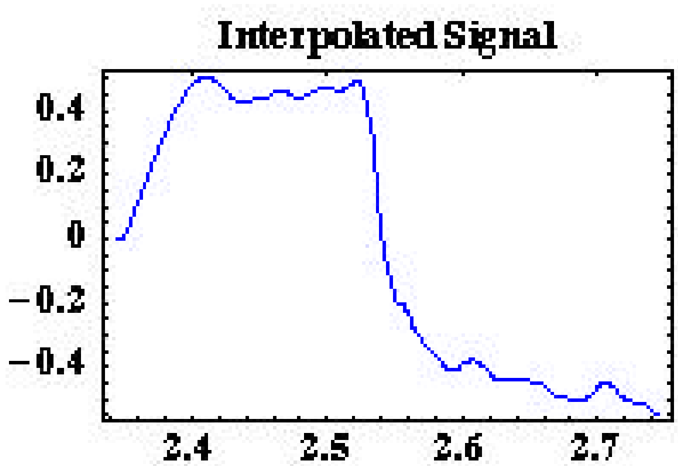


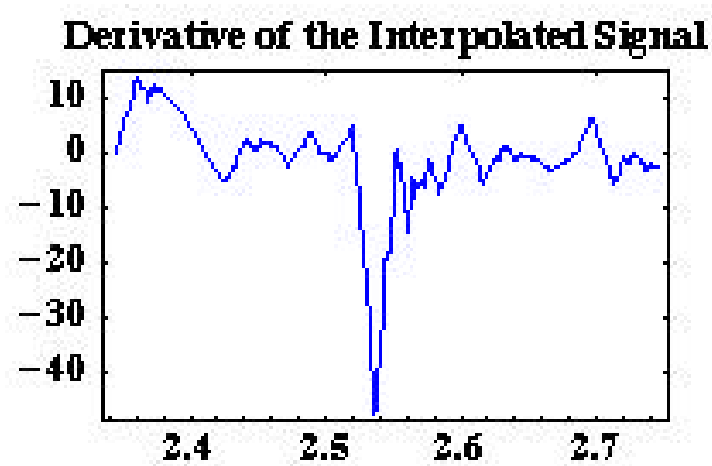


Reconstruction of Very Noisy Bolometer Energy & Power with 3 Lowest MRD Levels Using d5

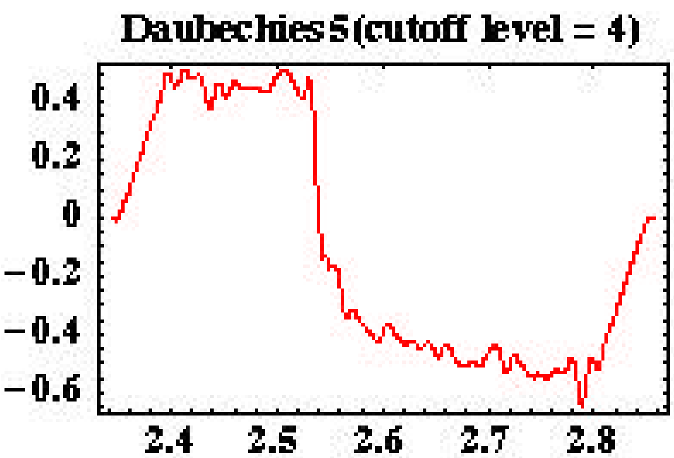


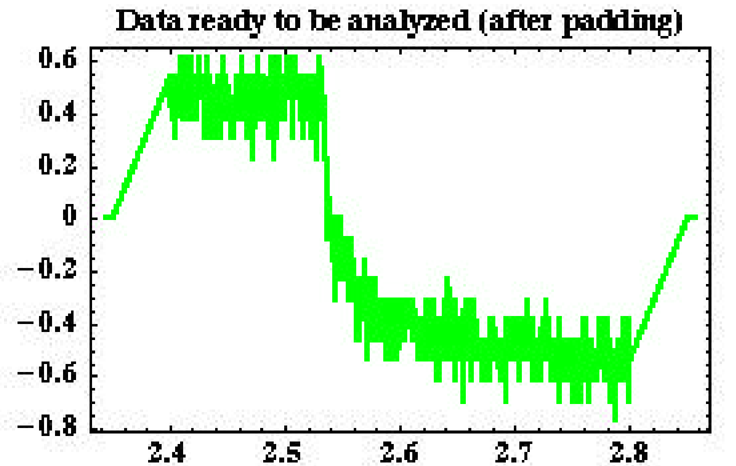


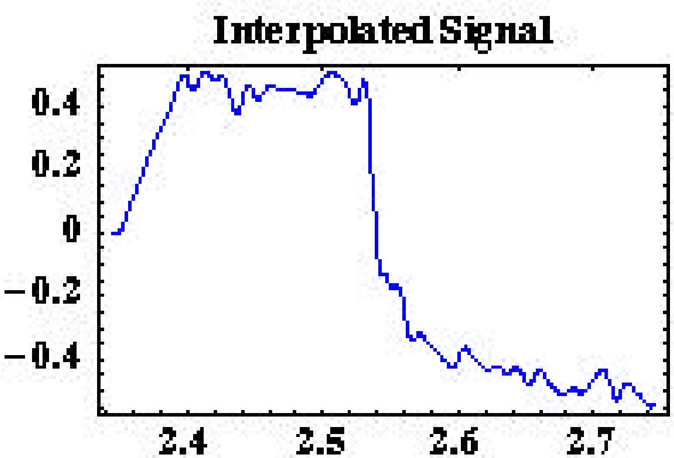


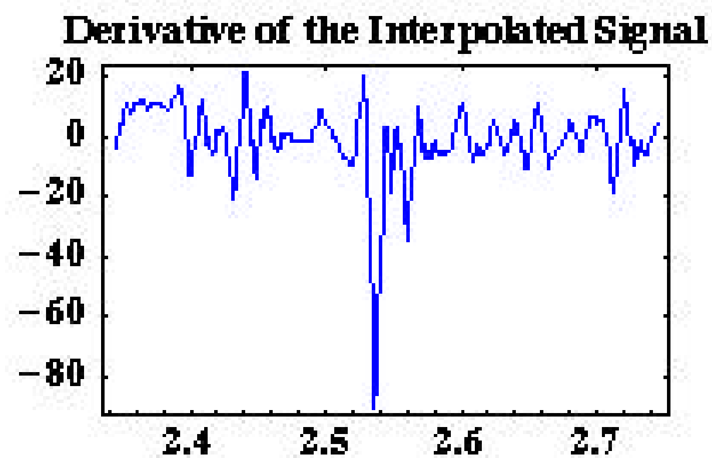


Reconstruction of Very Noisy Bolometer Energy & Power with 4 Lowest MRD Levels Using d5



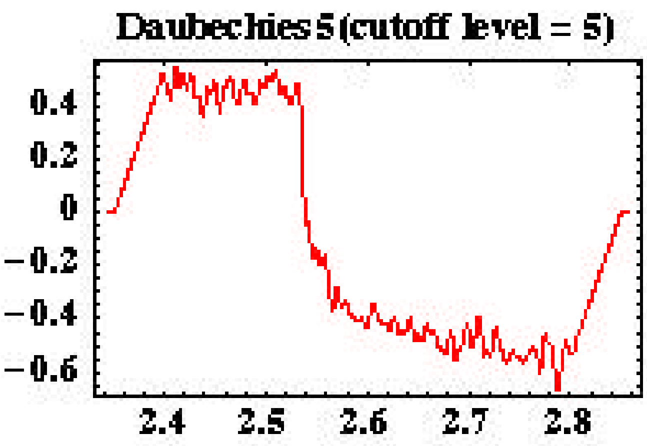


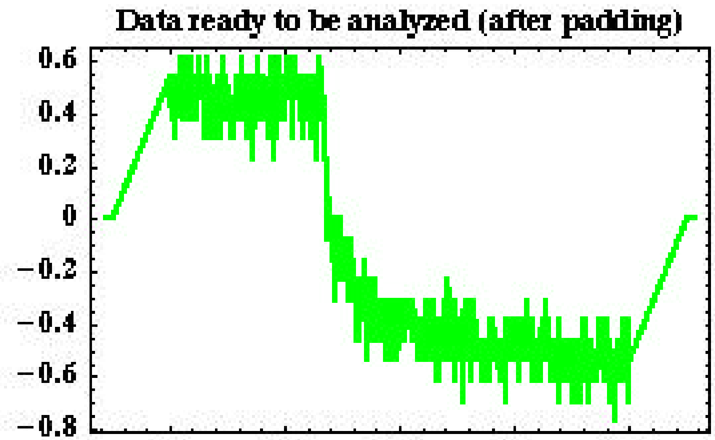


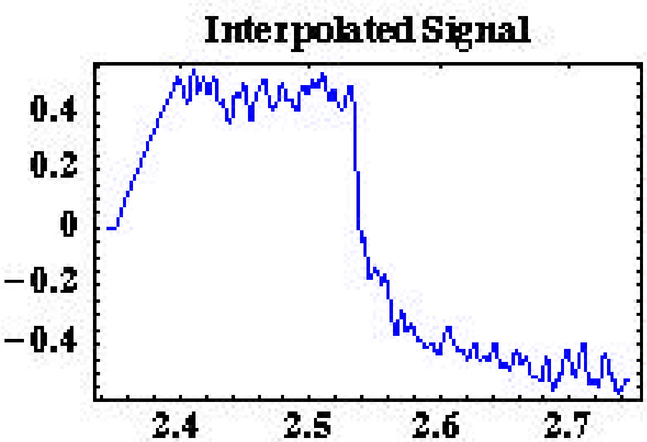


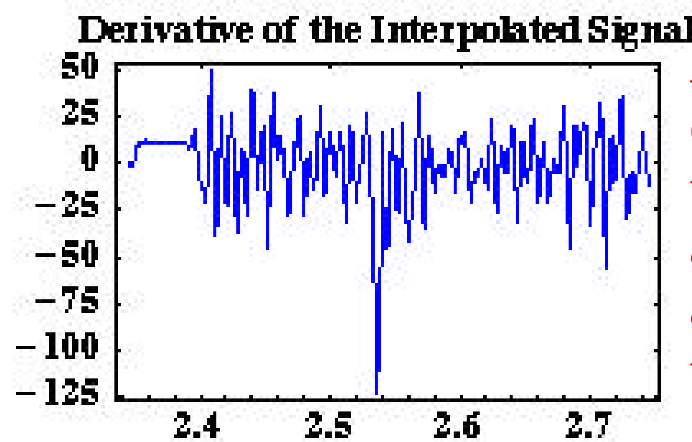
Reconstruction of Very Noisy Bolometer Energy & Power with 5 Lowest MRD Levels Using d5

Polymath Research Inc.





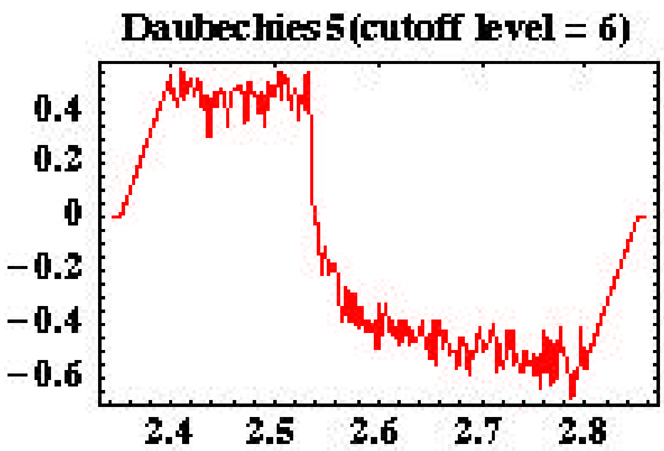


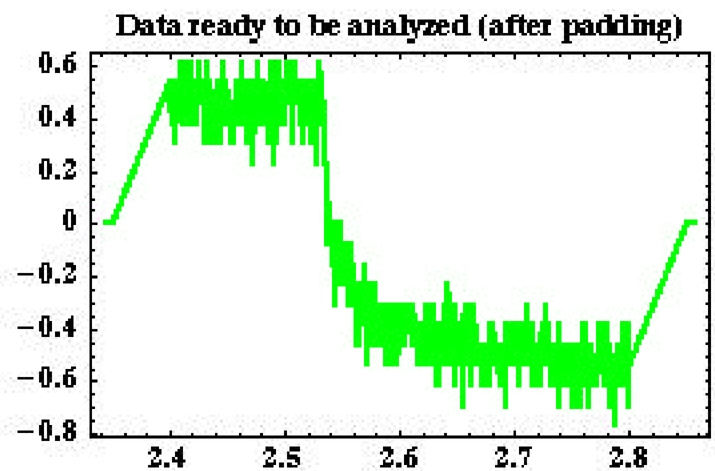


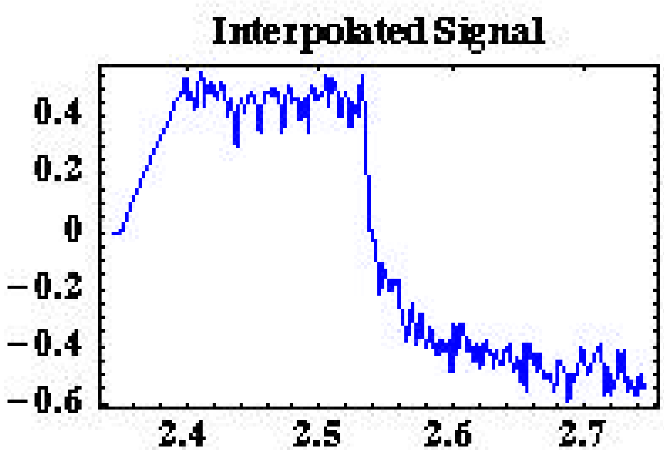
After this level, data is too noisy to differentiate and have anything left to differentiate!

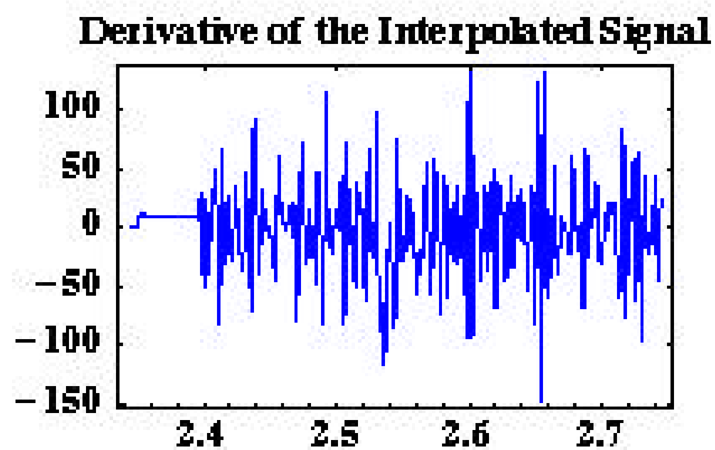
Reconstruction of Very Noisy Bolometer Energy & Power with 6 Lowest MRD Levels Using d5

Polymath Research Inc.









Conclusions:

- **Noisy data is a difficult thing to deal with!**
- **Need a lot better interpolator before power can be extracted from very noisy energy data.**
- **Better procedure is to do a low pass filter FIRST on the very noisy data and *then* do the mutiresolution decomposition on the remaining signal.**

Here Is the Very Noisy Data Low Pass Filtered with different Filter Widths and Cutoff Smoothnesses

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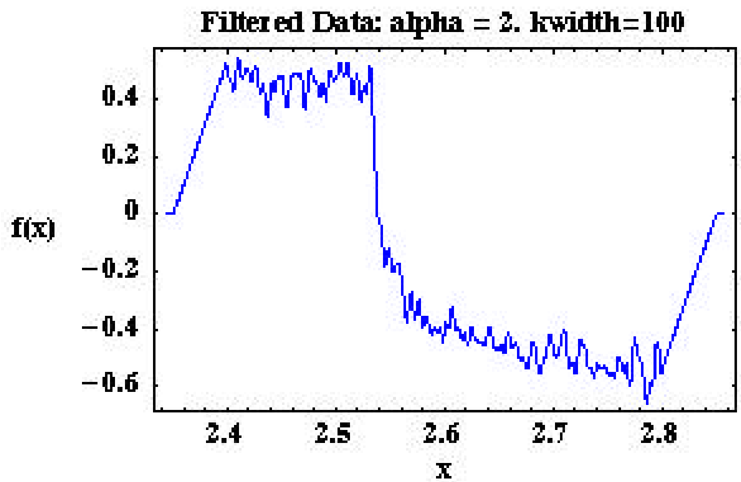
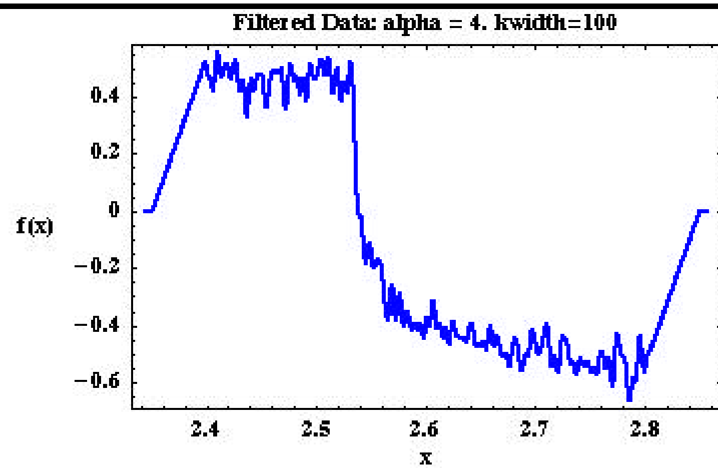
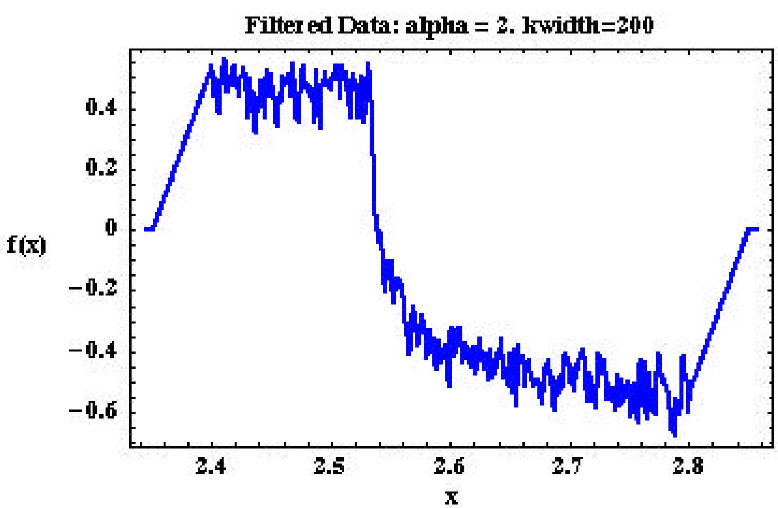
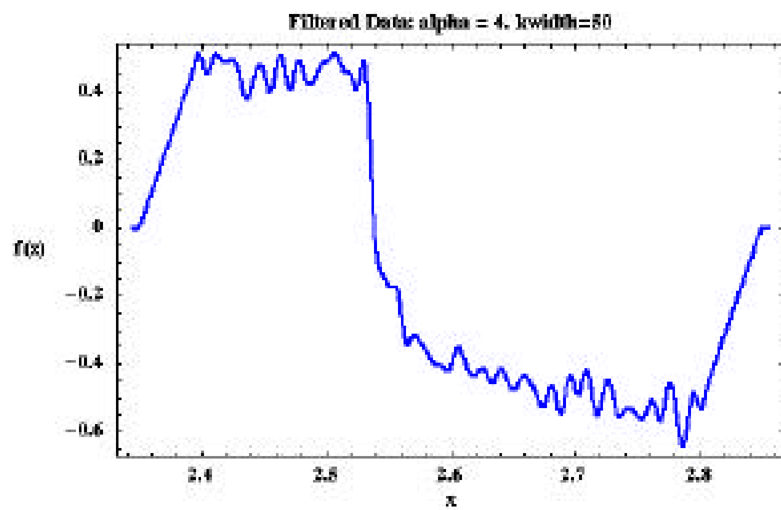




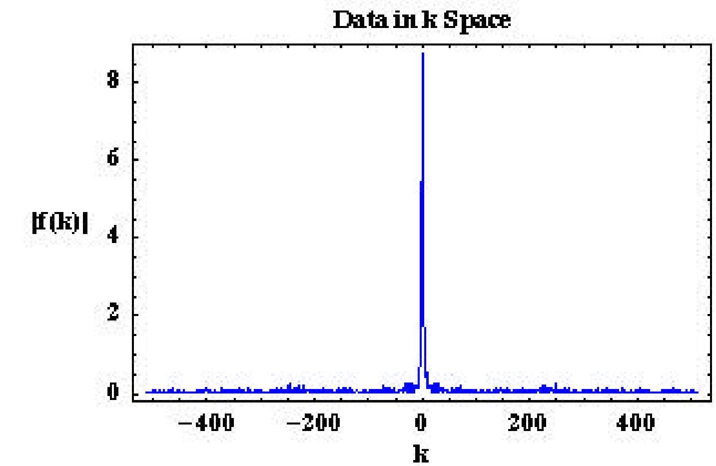
$$S(k) = \exp\left[-\left(\frac{k}{k_{width}}\right)^{2\alpha}\right]$$

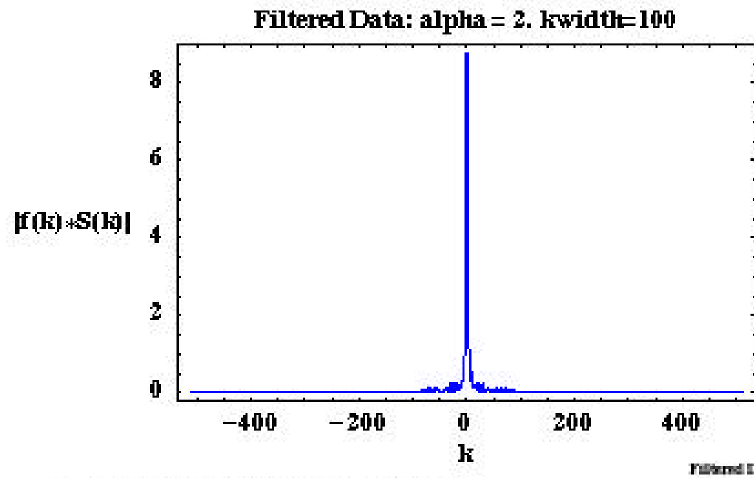


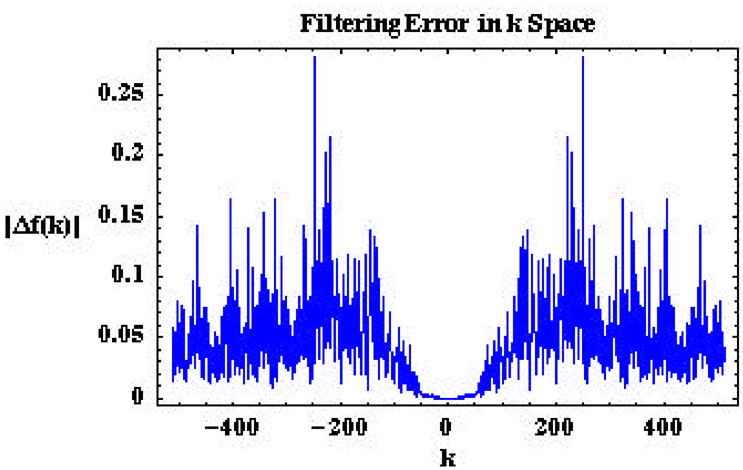


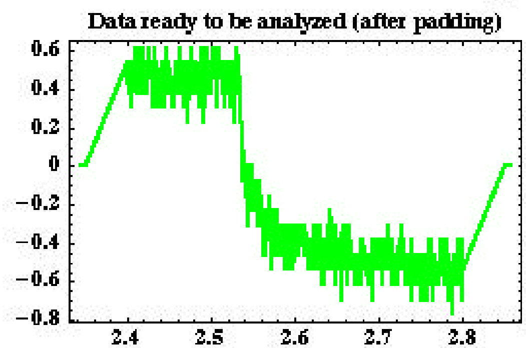
The Filter Is a Super-Gaussian in k space with width k_{width} = 100 and smoothness exponent 2 α = 4

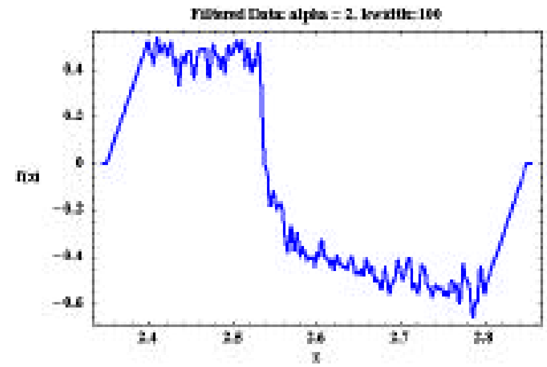
Polymath Research Inc.







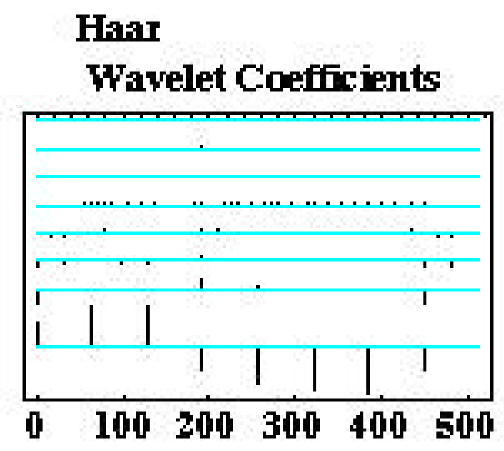


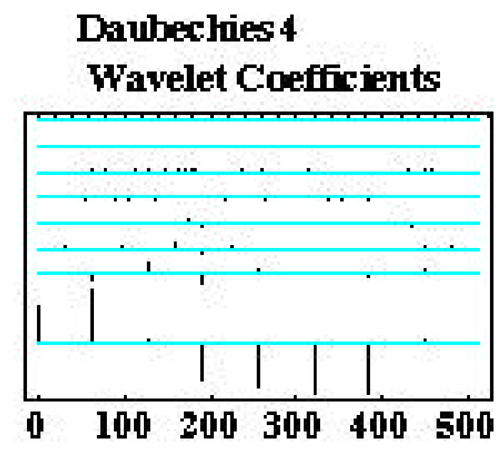


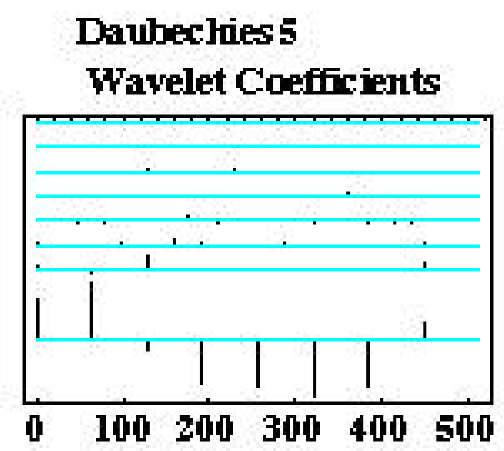
$$S(k) = \exp\left[-\left(\frac{k}{k_{width}}\right)^{2\alpha}\right]$$

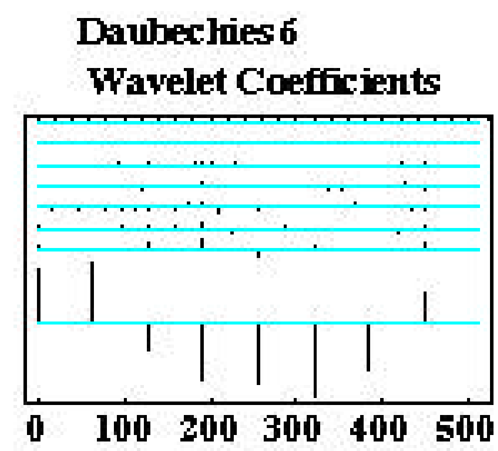
Low Pass Filtered Very Noisy Bolometer Energy Signal MRD Coefficients (alpha=2, kwidth=100)

Polymath Research Inc.



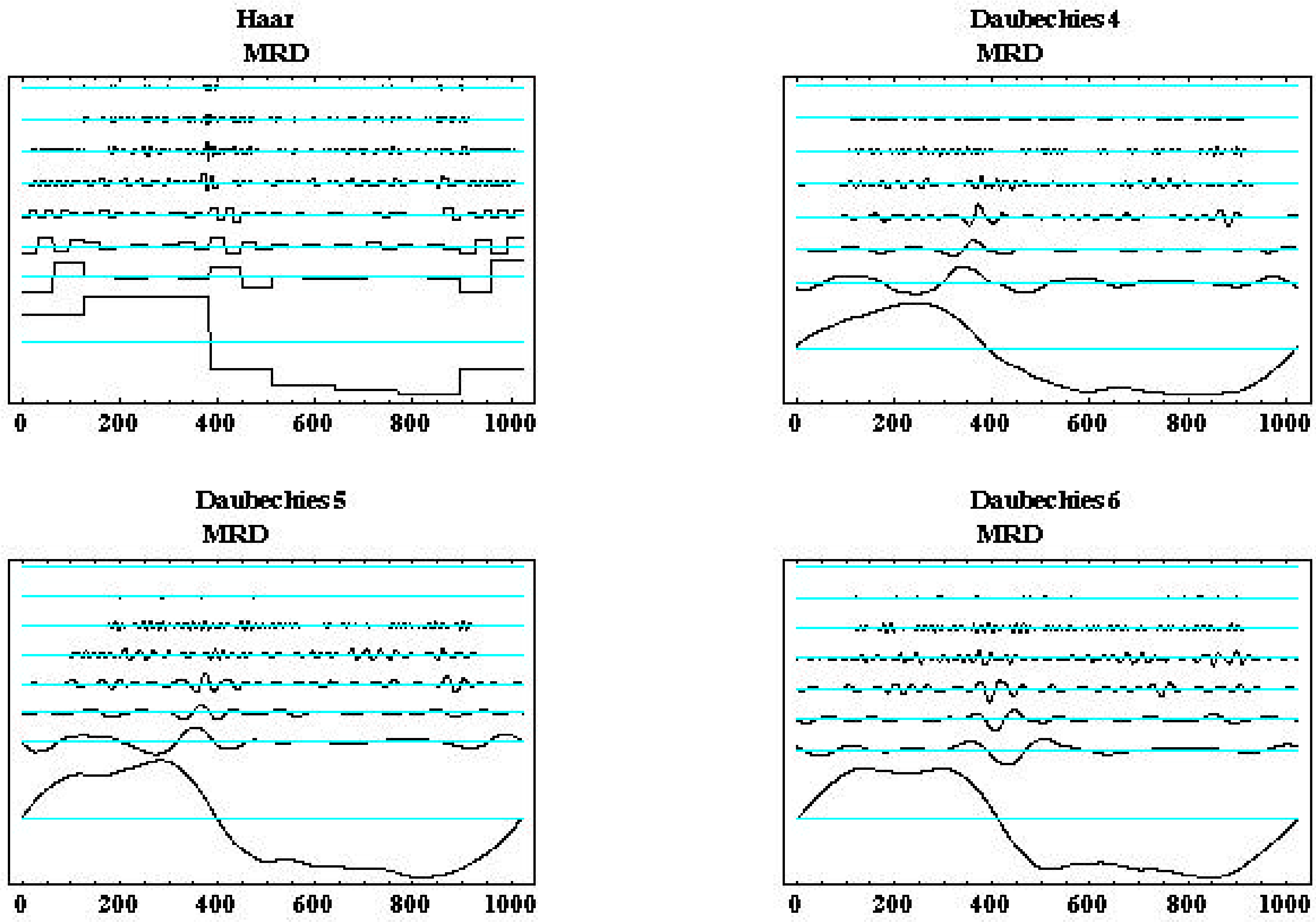




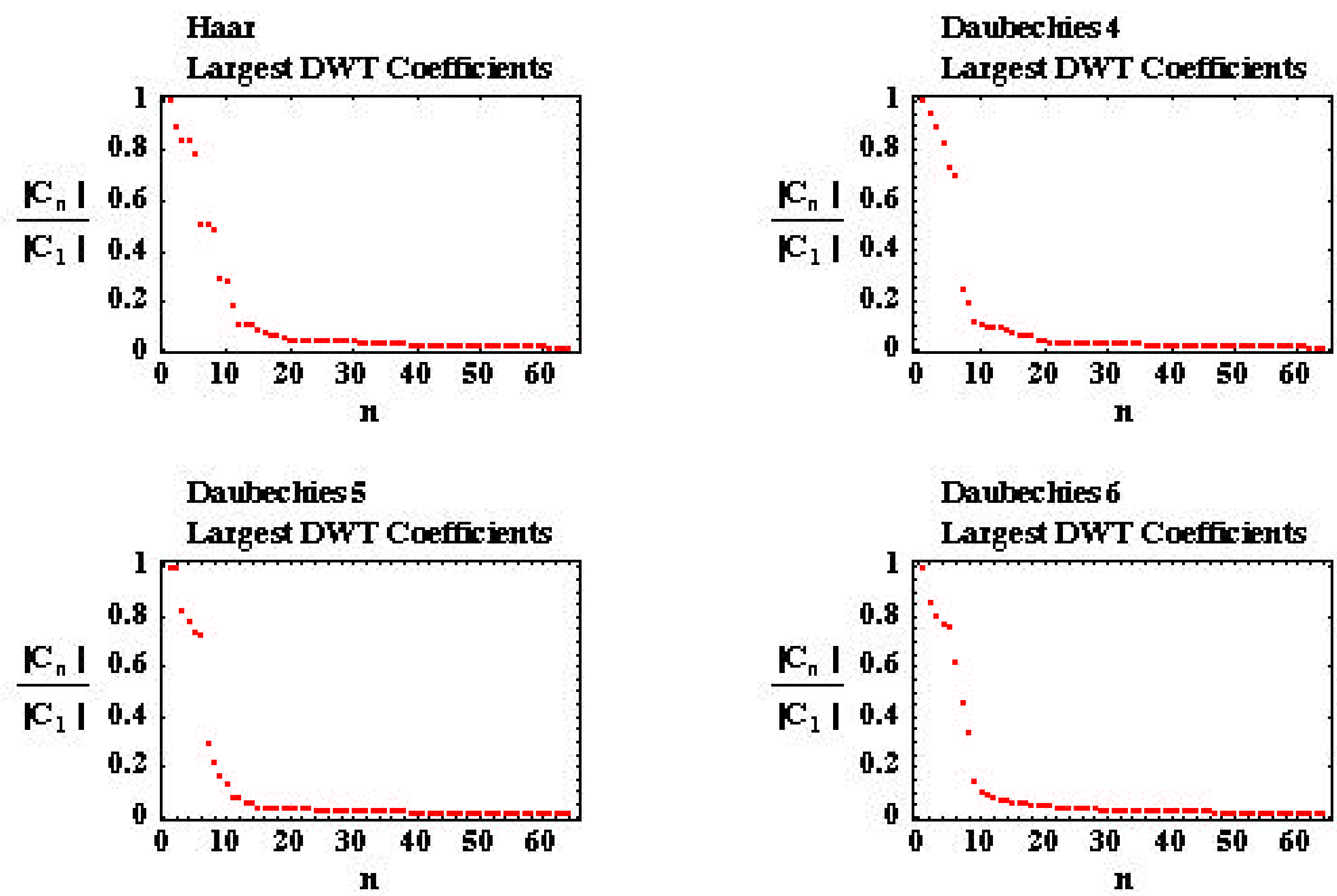


Low Pass Filtered Very Noisy Bolometer Energy Signal MRD

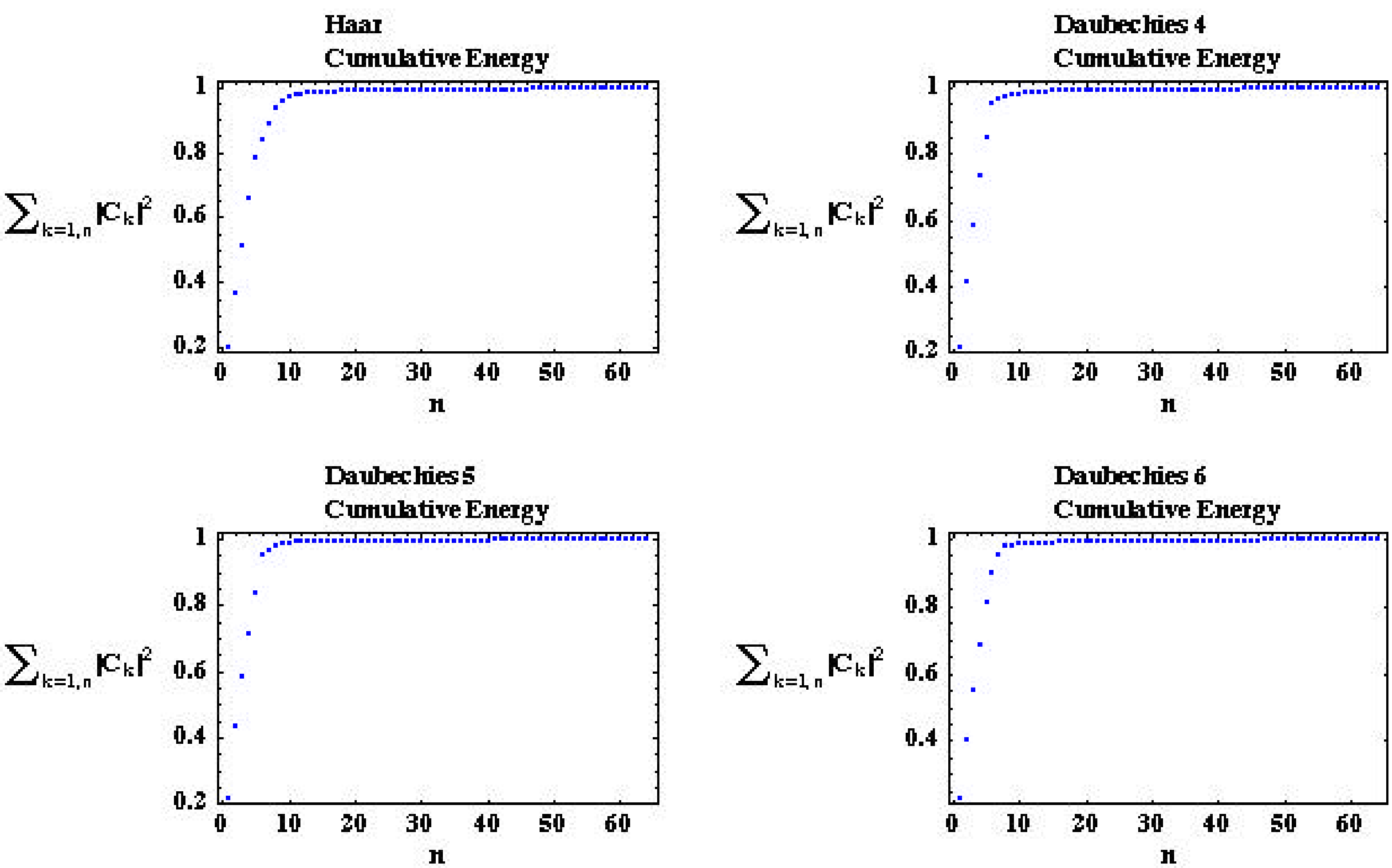
Polymath Research Inc.



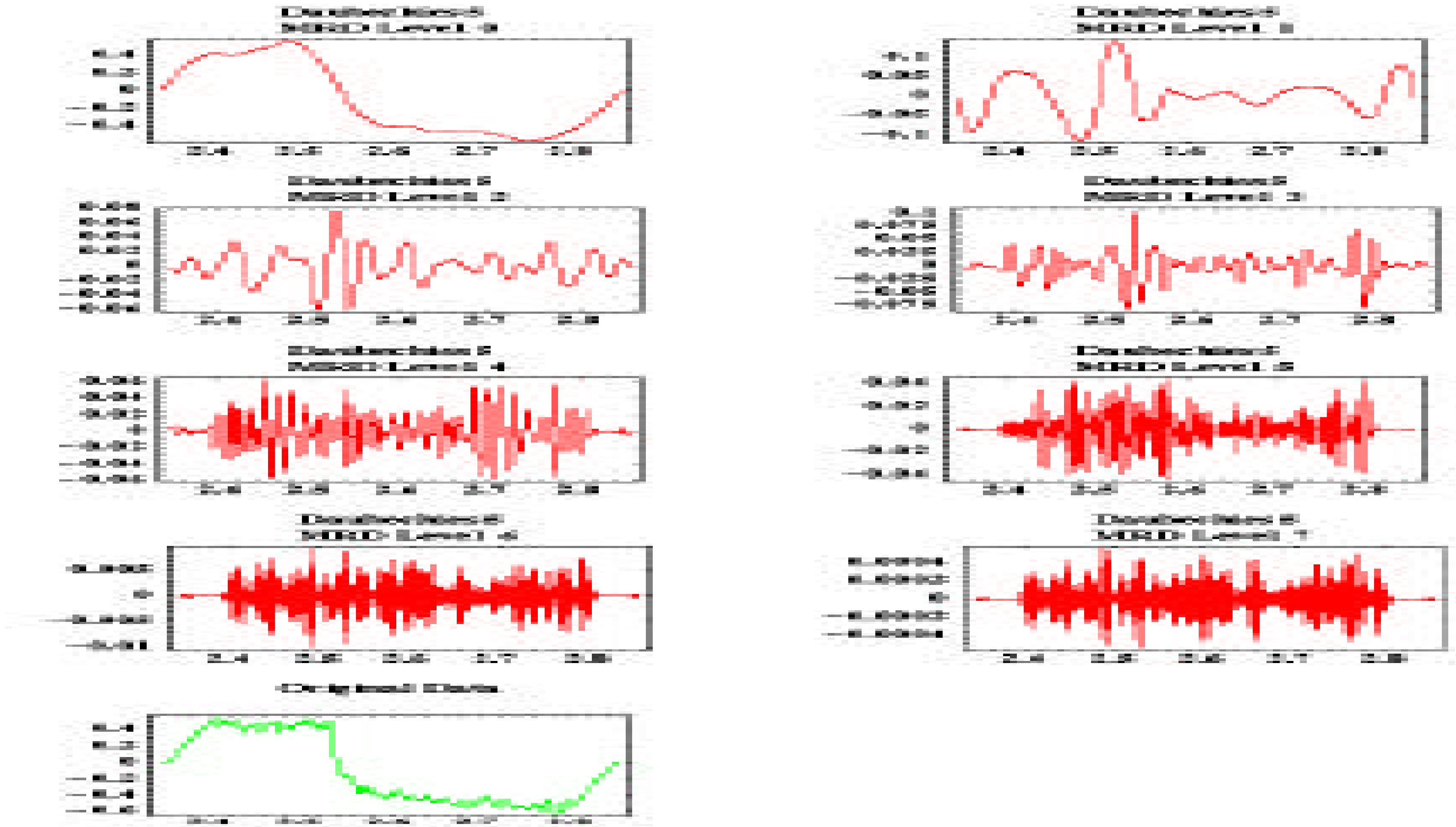
Decay of Largest Coefficients in the MRD of LPF Very Noisy Bolometer Energy Data



Cummulative Energy in Coefficient Space for LPF Very Noisy Bolometer Energy Data

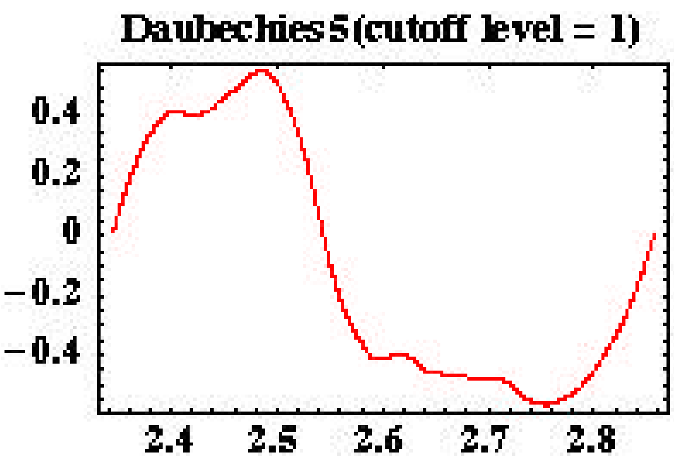


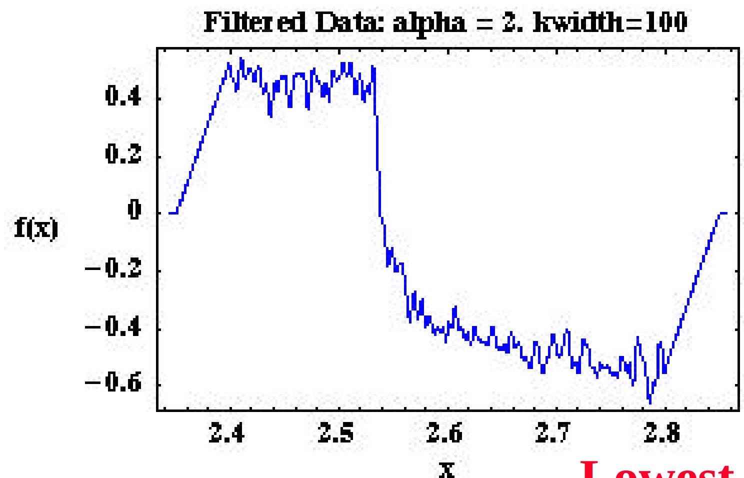
D5 MRD of LPF Very Noisy Bolometer Energy Data



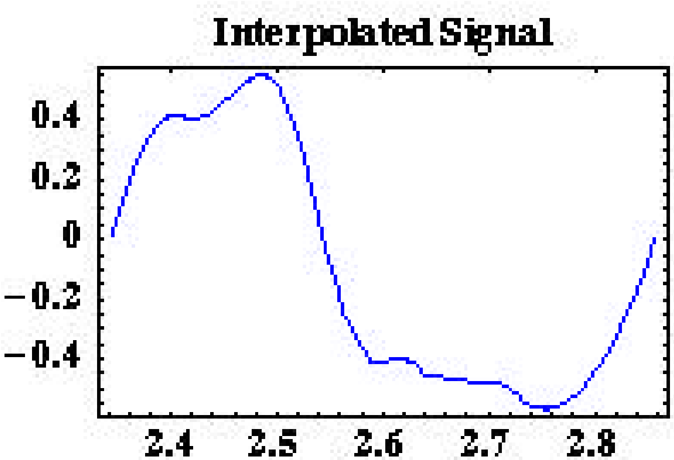
With Filter Roll off Smoothness alpha=2 & k_width=100, We Can Get the Power off Energy Lineouts

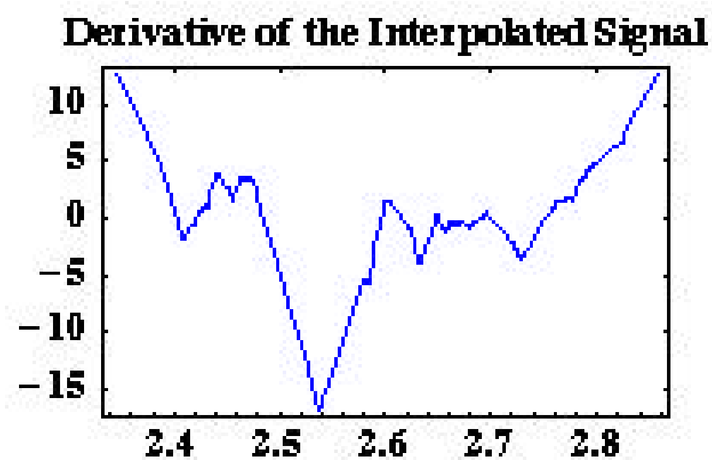
Polymath Research Inc.



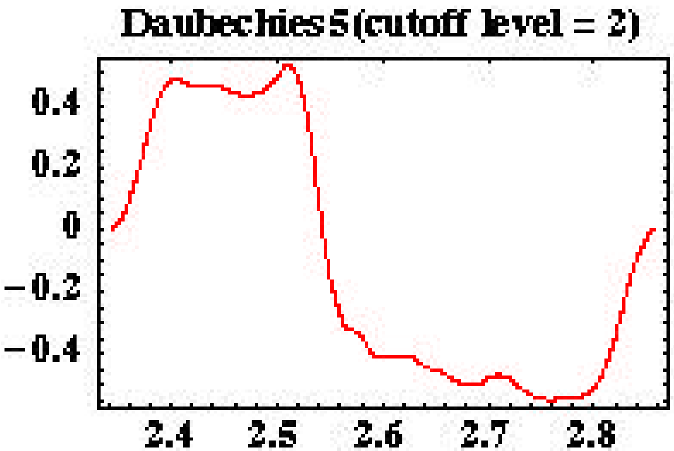


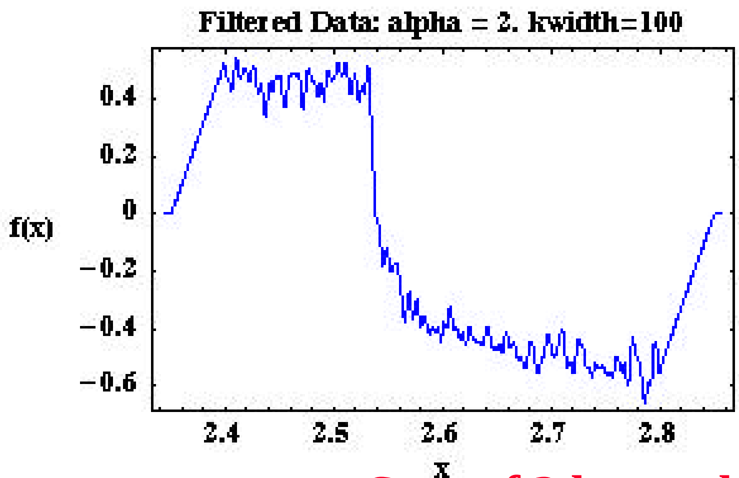
Lowest Level of MRD



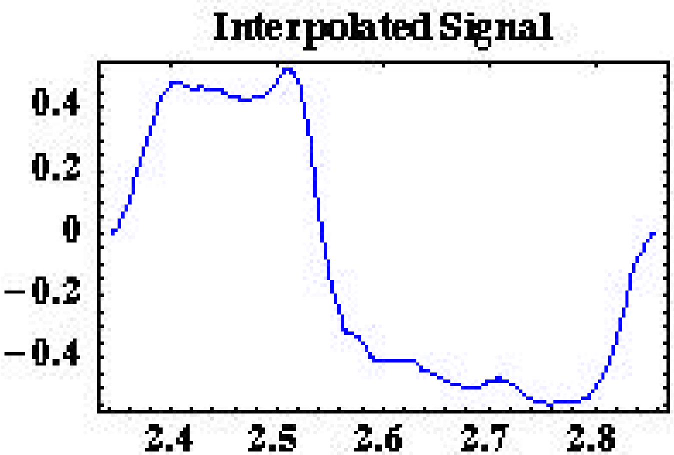


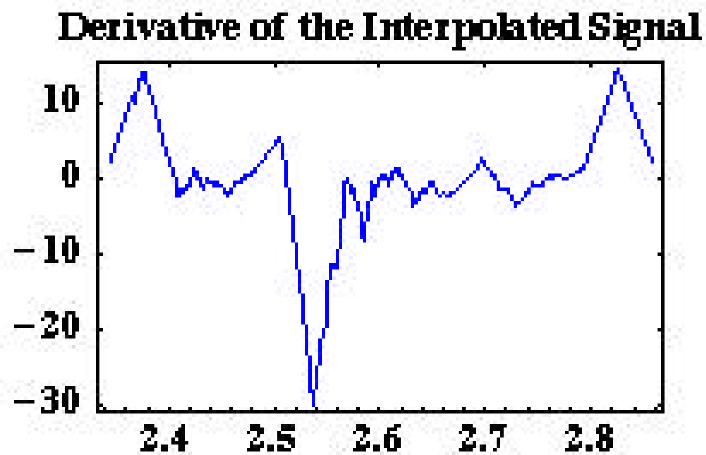
With Filter Roll off Smoothness alpha=2 & k_width=100, We Can Get the Power off Energy Lineouts





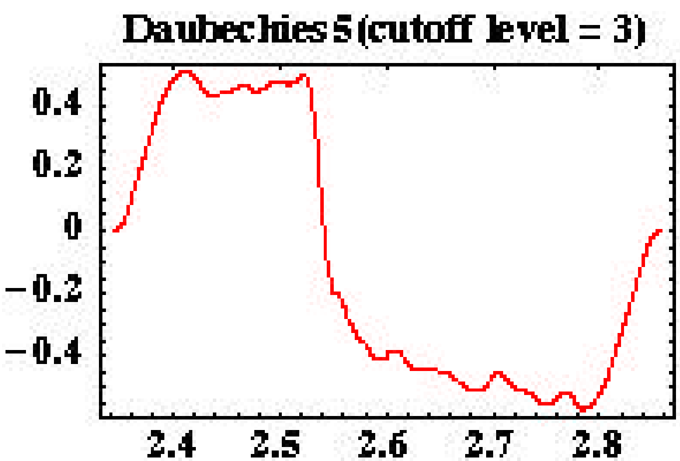
Sum of 2 lowest levels of MRD

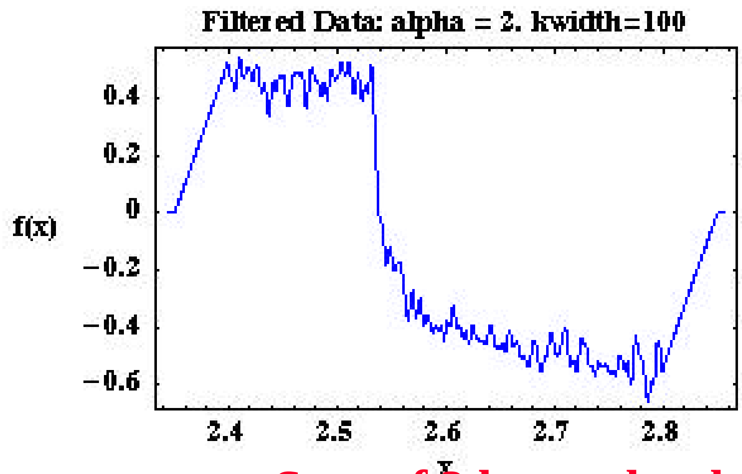




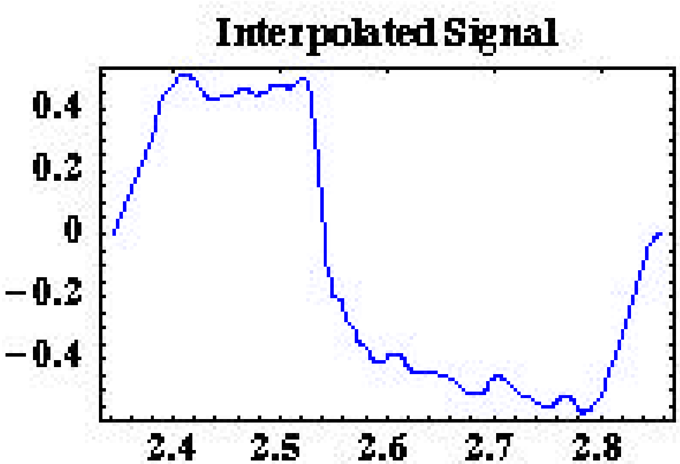
With Filter Roll off Smoothness alpha=2 & k_width=100, We Can Get the Power off Energy Lineouts

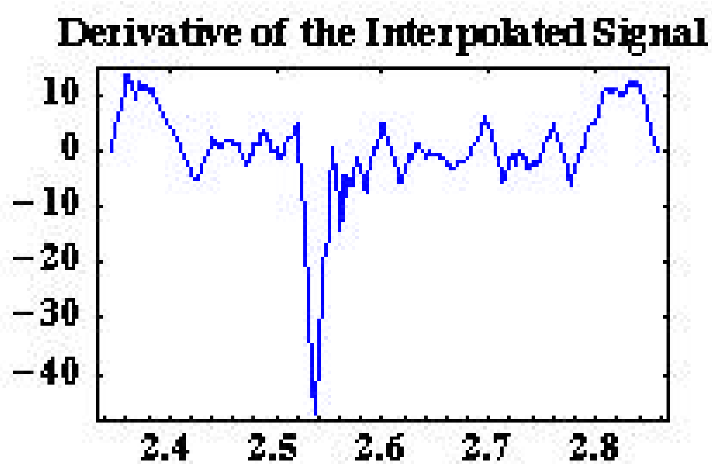
Polymath Research Inc.



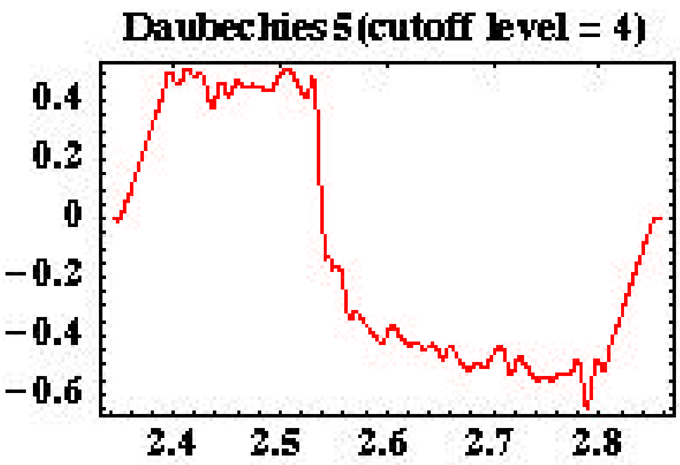


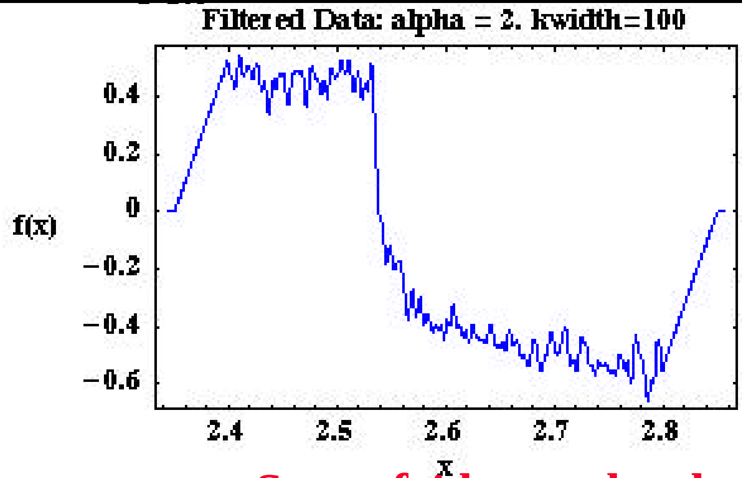
Sum of 3 lowest levels of MRD



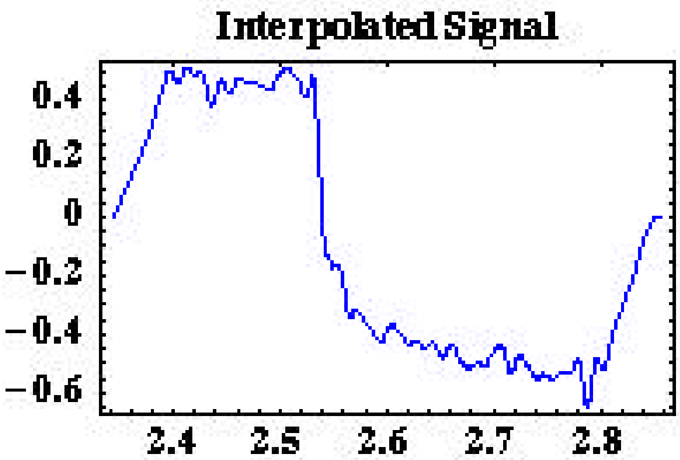


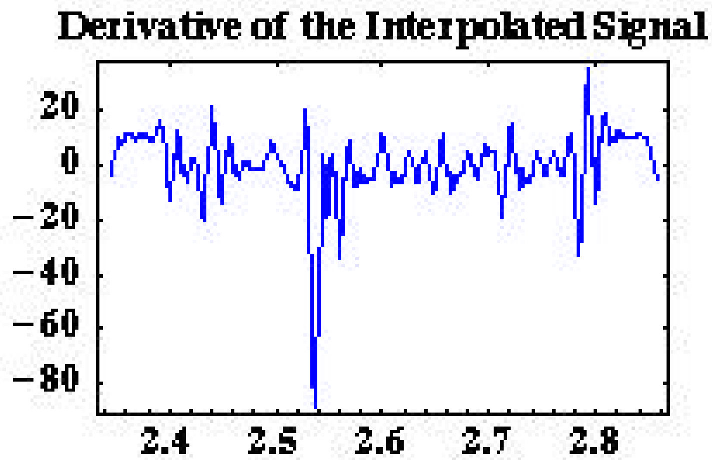
With Filter Roll off Smoothness alpha=2 & k_width=100, We Can Get the Power off Energy Lineouts



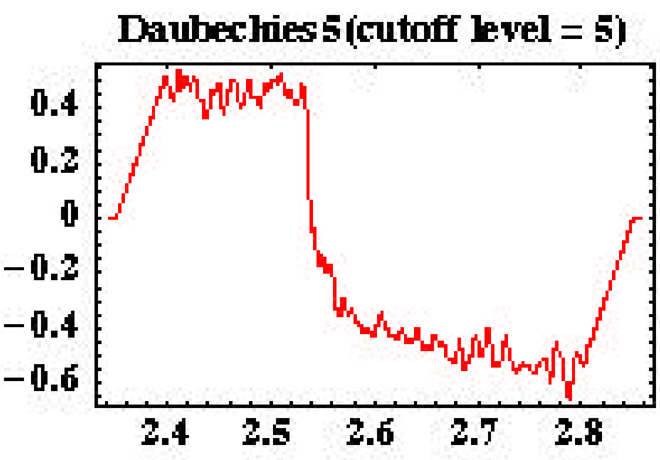


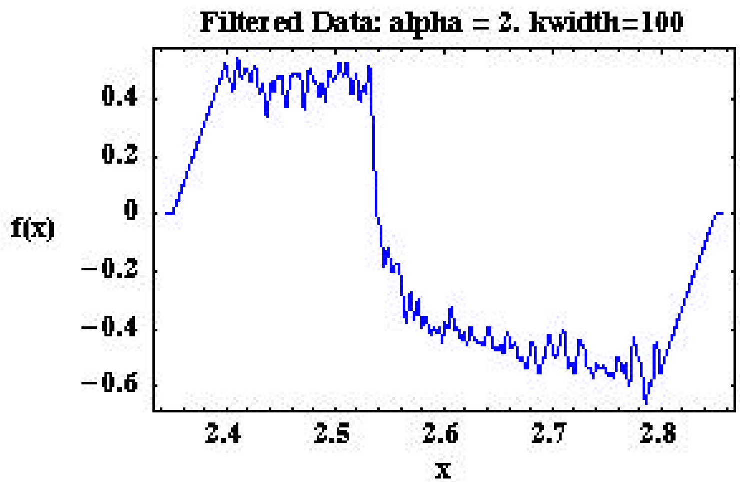
Sum of 4 lowest levels of MRD



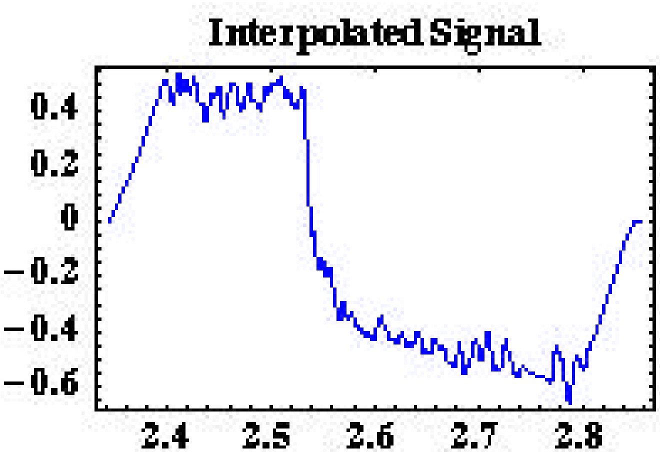


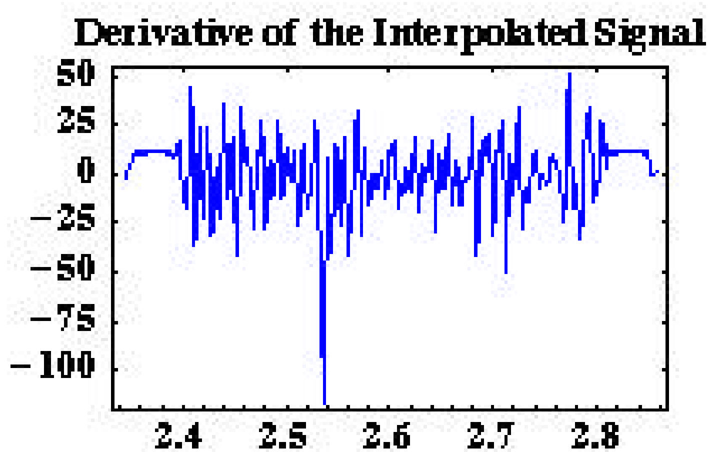
With Filter Roll off Smoothness alpha=2 & k_width=100, We Can Get the Power off Energy Lineouts



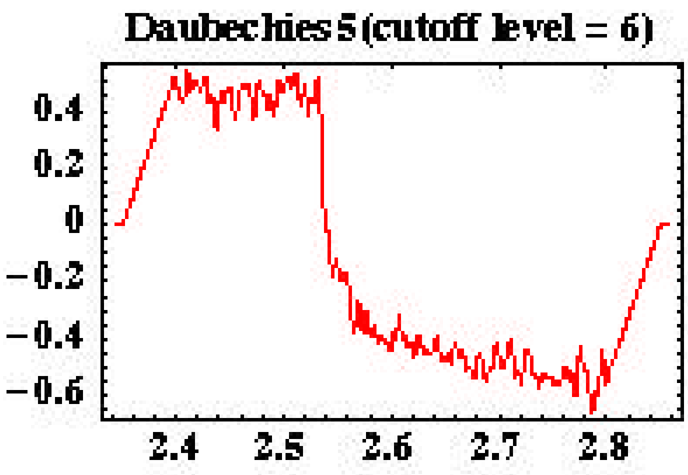


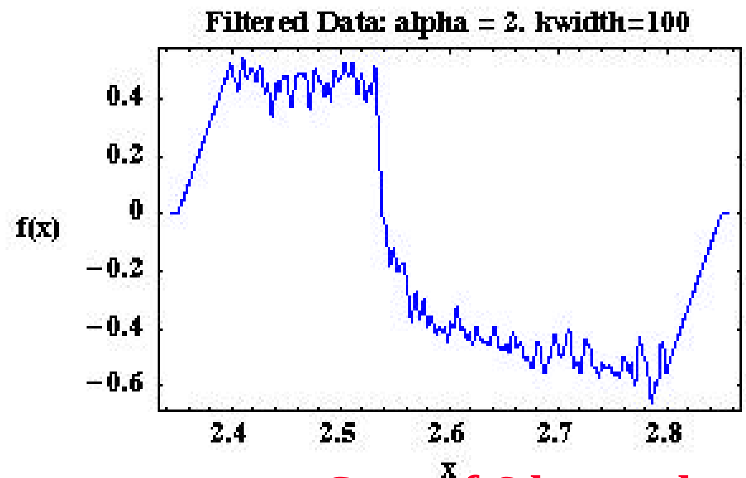
Sum of 5 lowest levels of MRD



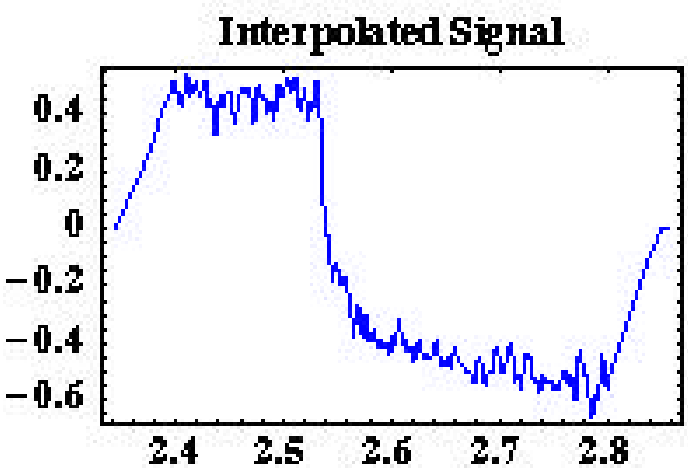


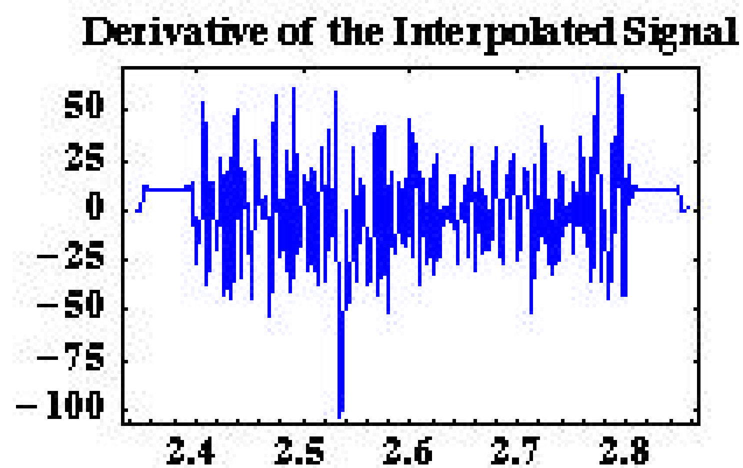
With Filter Roll off Smoothness alpha=2 & k_width=100, We Can Get the Power off Energy Lineouts





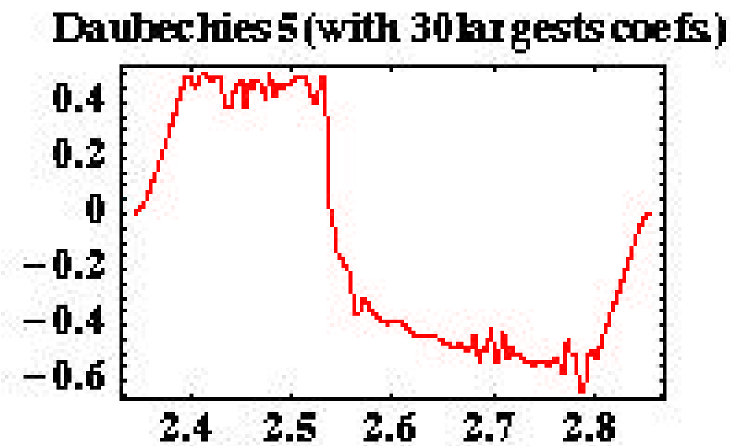
Sum of 6 lowest levels of MRD

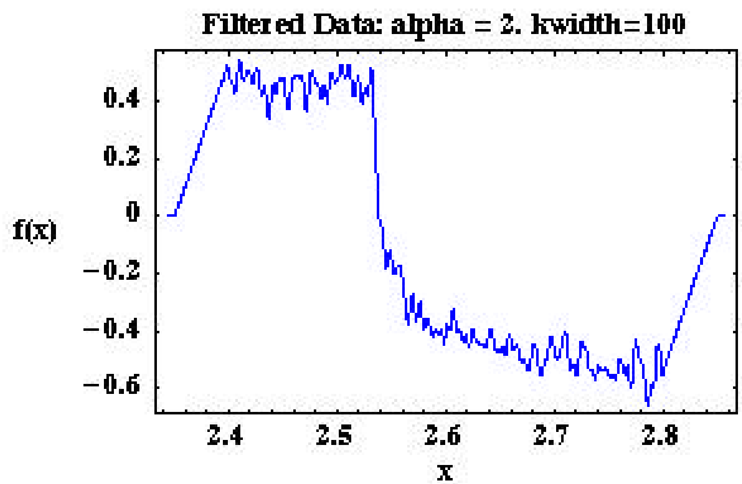


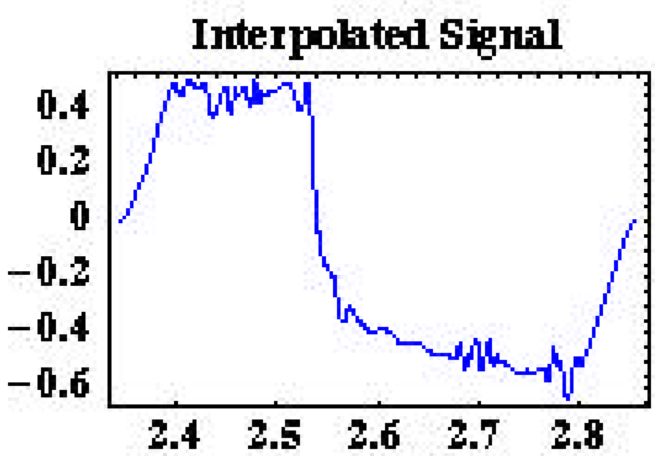


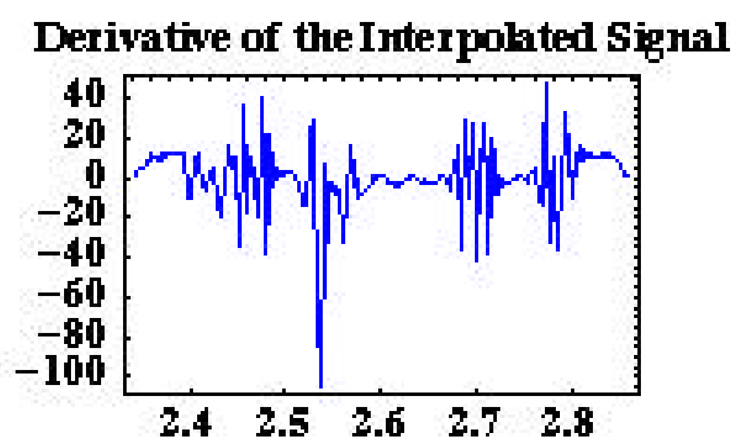
30 Largest Wavelet Coefficient Thresholding Using D5 on LPF Very Noisy Bolometer Energy

Polymath Research Inc.

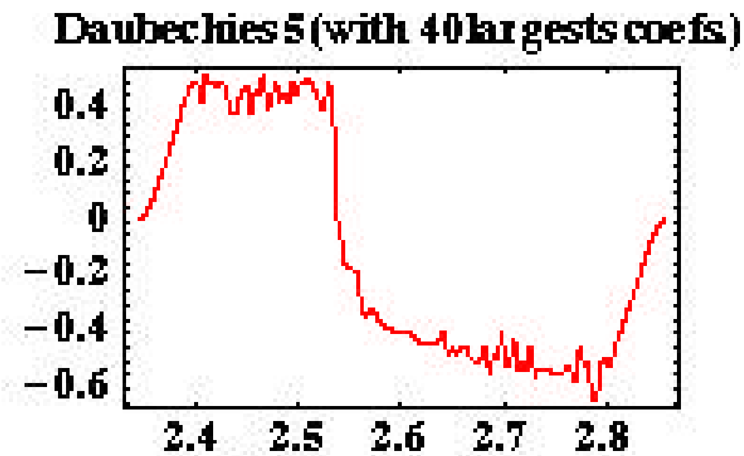


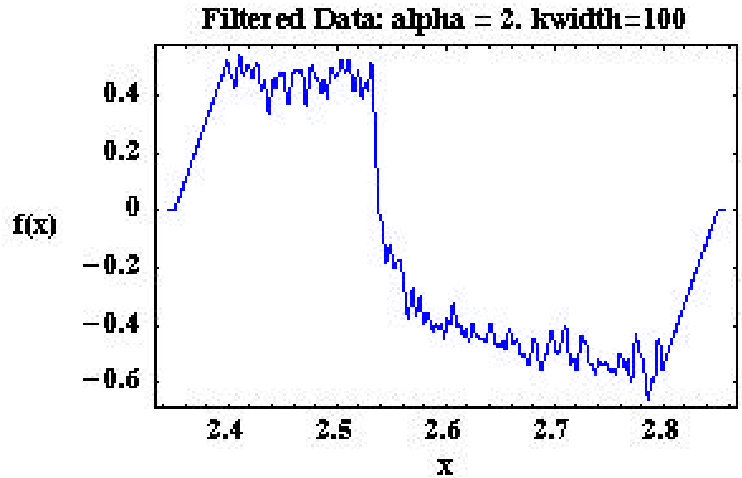


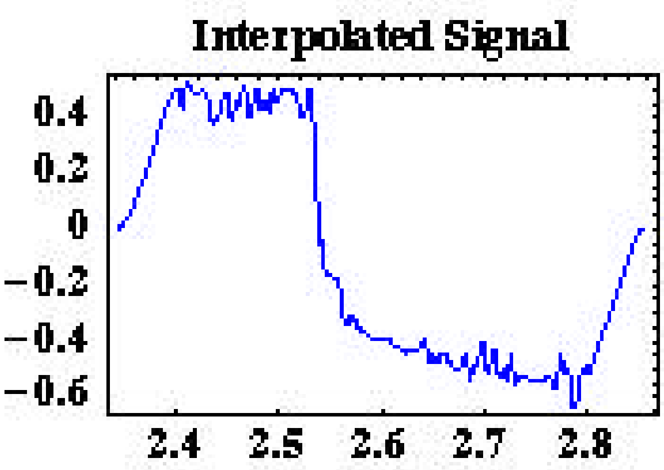


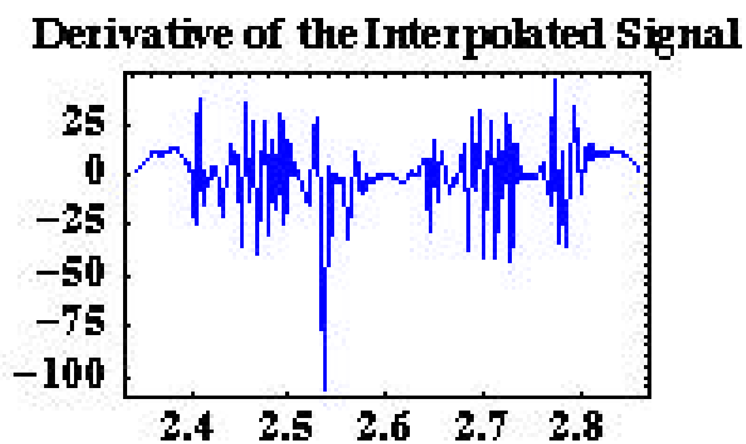


40 Largest Wavelet Coefficient Thresholding Using D5 on LPF Very Noisy Bolometer Energy

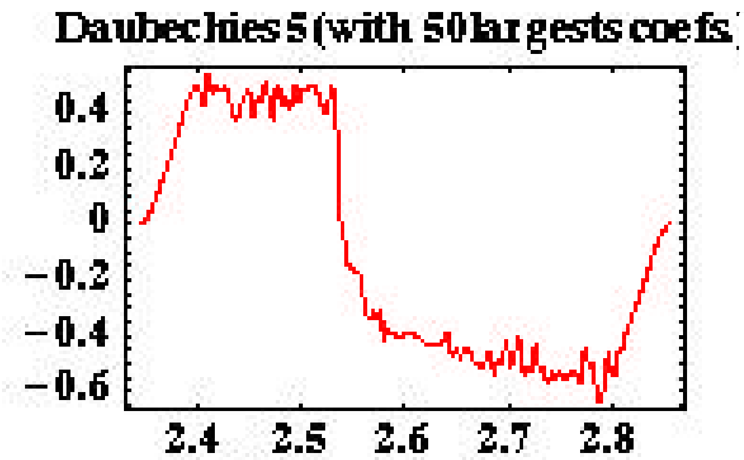


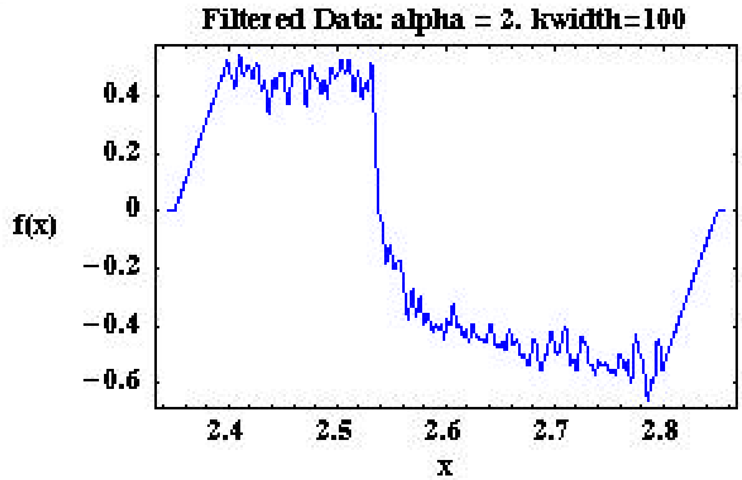


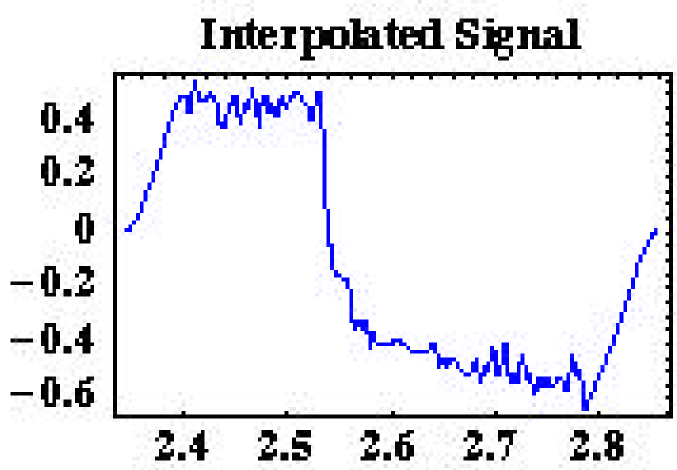


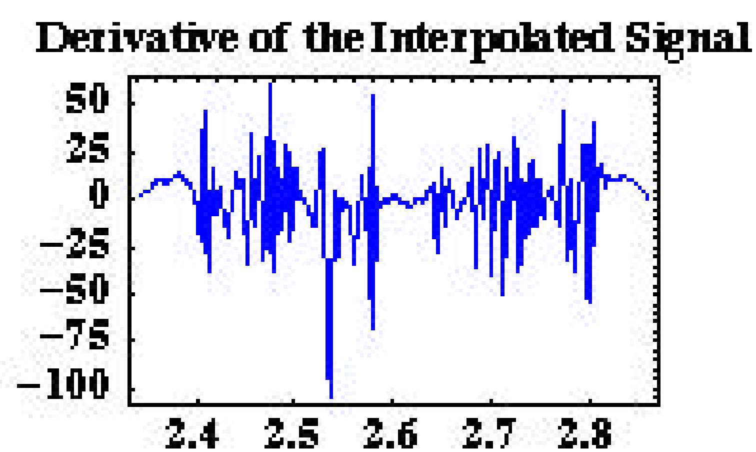


50 Largest Wavelet Coefficient Thresholding Using D5 on LPF Very Noisy Bolometer Energy

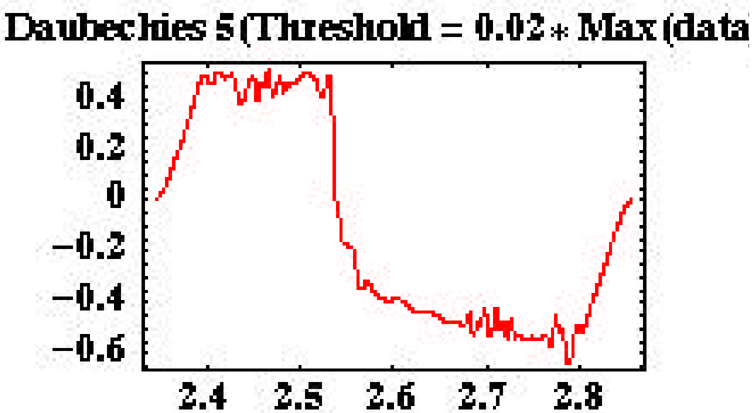


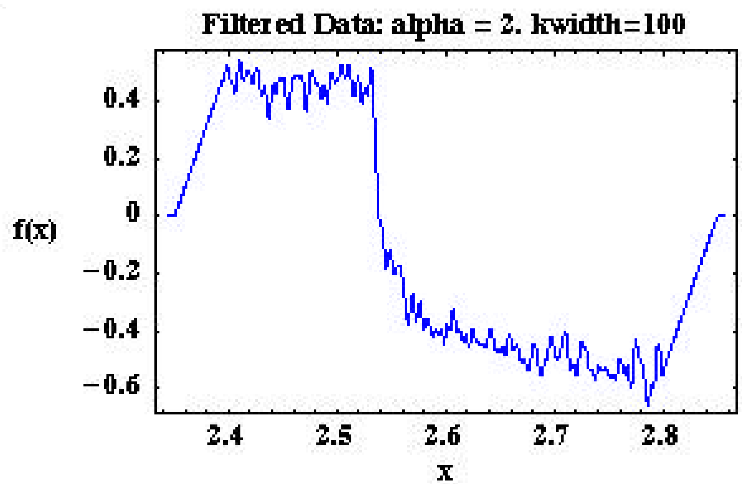


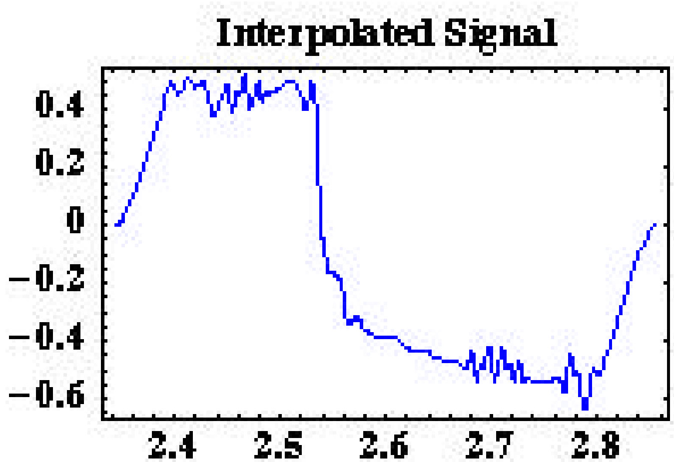


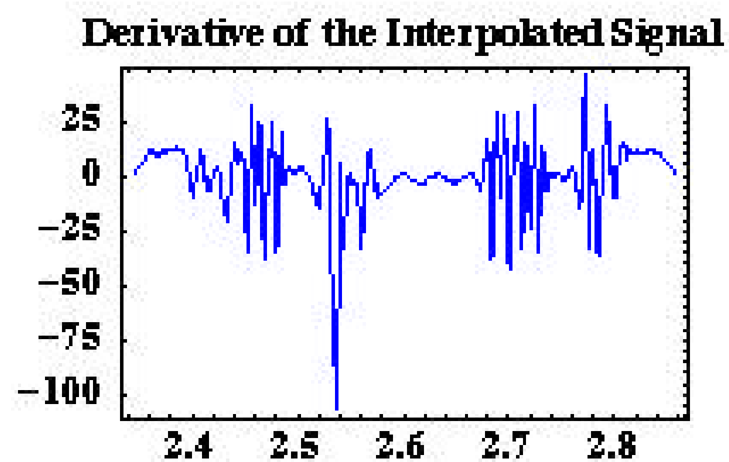


Tthresholding by Keeping Coeffieintc Greater than 2% of the Largest Coefficient Also Works with D5









Overall Conclusions

- If the data is really noisy as in some of the bolometer energy lineouts, trying to extract power curves from that is perilous and not always possible.
- Much denoising is required and Fourier and Wavelet techniques can work in a complementary fashion.
- Multiresolution decomposition (fast discrete wavelet transform) allows a clear understanding of where in space (or time) certain features are prominent and on what scales.
- Spiky features which are spatially (or temporally) highly localized can very efficiently be picked up by wavelet largest coefficient thresholding. The largest coefficients will home in on those features with few of them needed no matter how much noise accompanies those spikes.
- Level decomposition allows us to smooth out the data and keep features we want and discard the rest in the opposite limit.

Rayleigh Taylor Turbulence in Plane View: Photograph of the Experiment

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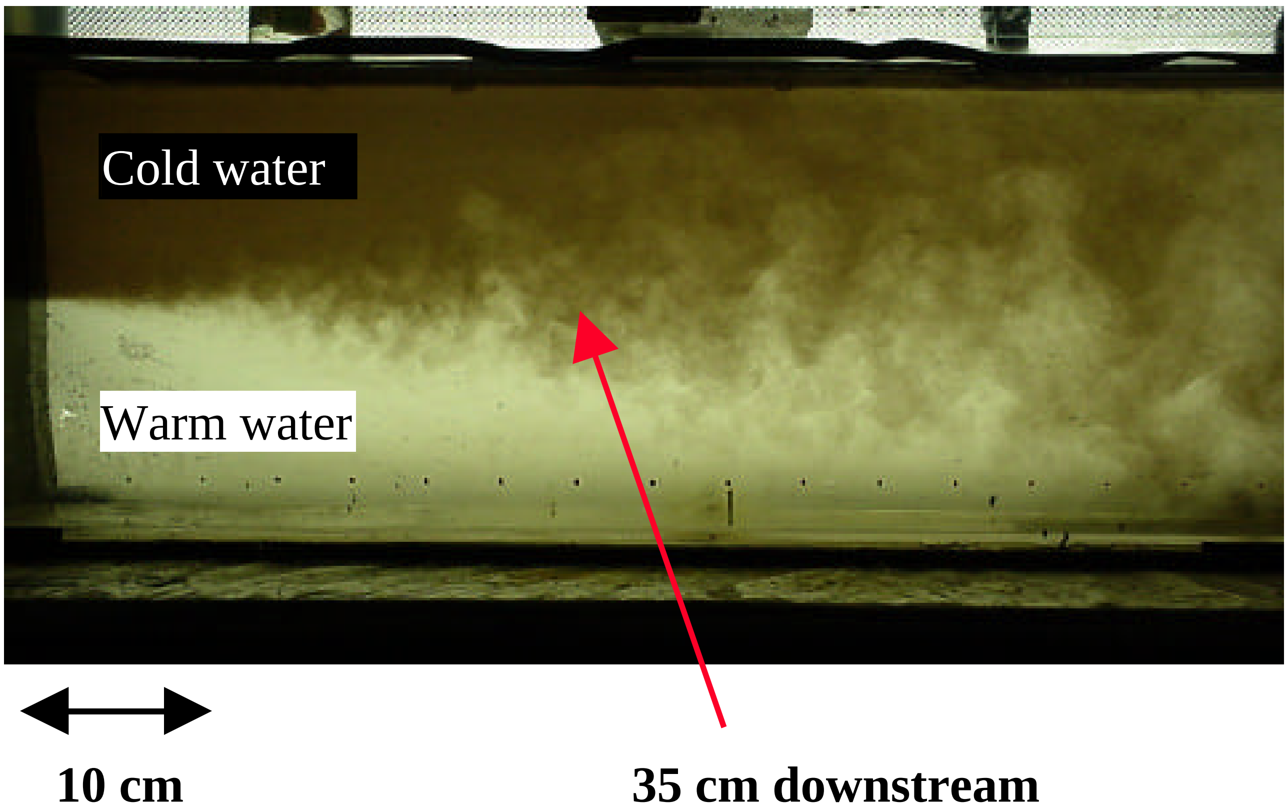


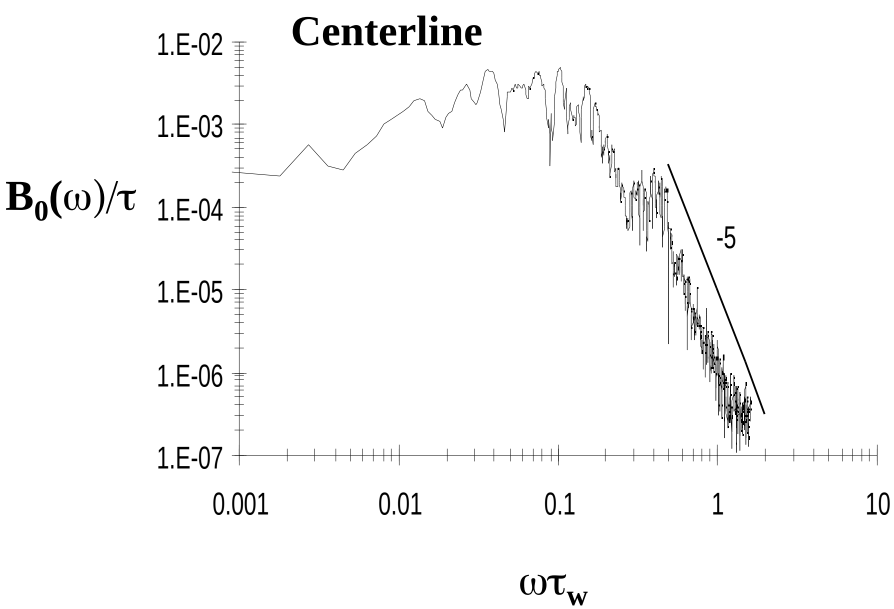
Atwood # = 10^{-3}

$\Delta T = 5^{o}C$

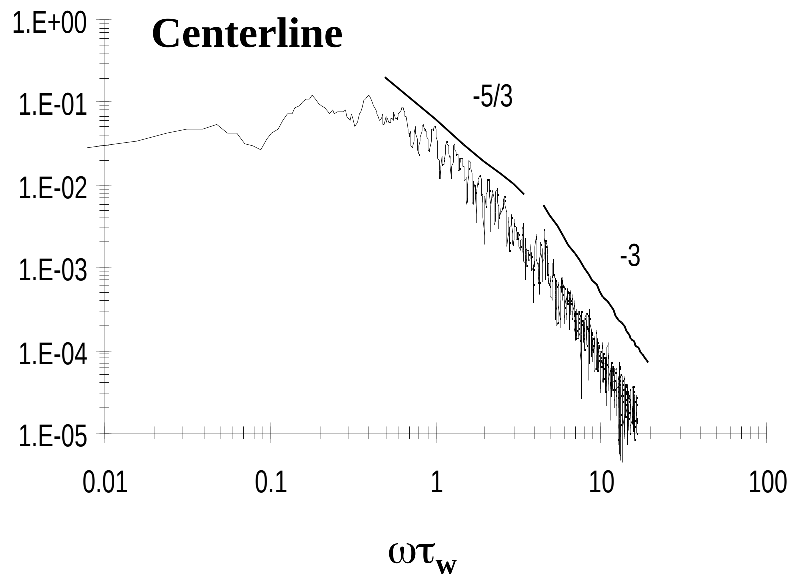
U = 4 cm/s

Density fluctuation power spectra

2.4 cm downstream



30 cm downstream



Experimental Observations

- Cold water runs over warm water at the same speed, and downstream mixes due to buoyancy. The downstream distance is the "time" evolution of the mix.
- The photograph shows a rapidly expanding mix (quadratic growth of the edges), and increasing length-scales downstream, with lots of internal small scale structures.
- The centerline single-point density fluctuation power spectrum has significantly developed from the start (2.4 cm), and shows no apparent relation to the late-time (30 cm) profile.
- The power spectra show no "local" (in time) structure, so we don't know if there is any embedded information at late time from the early time.

Raw Thermocouple RT Strong Mix Data (30 cm Downstream, theta ~ 0.71) from Texas A&M

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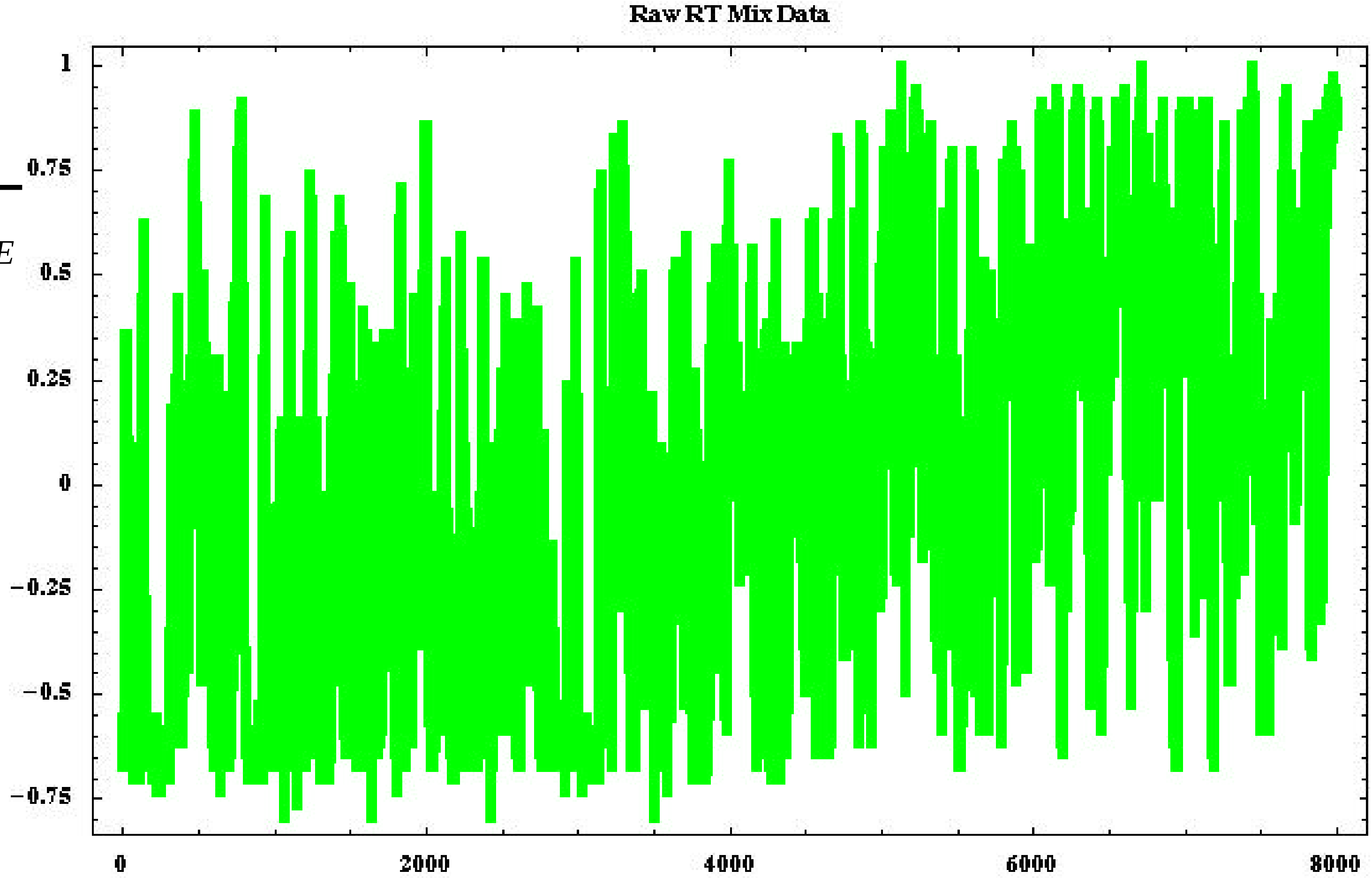
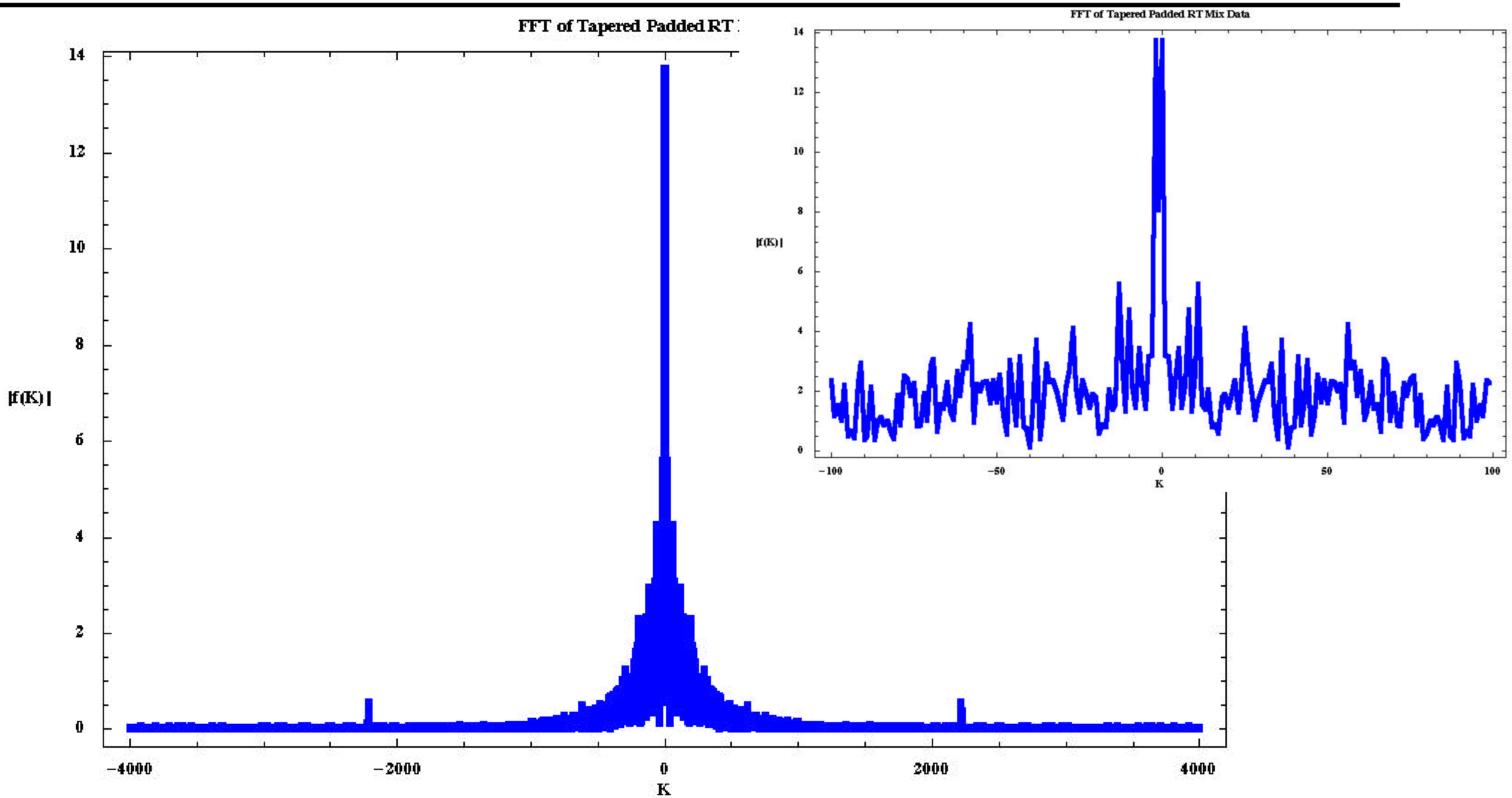
$$\frac{T - T_{AVE}}{T_{MAX} - T_{AVE}}$$



Time, arb. units (Delta t=0.012 sec, Sampling Rate = 85 Hz)

The Fourier Transform of the RT Mix Data



Level by Level Decomposition of the RT Mix Data Using Daub5 WLTs

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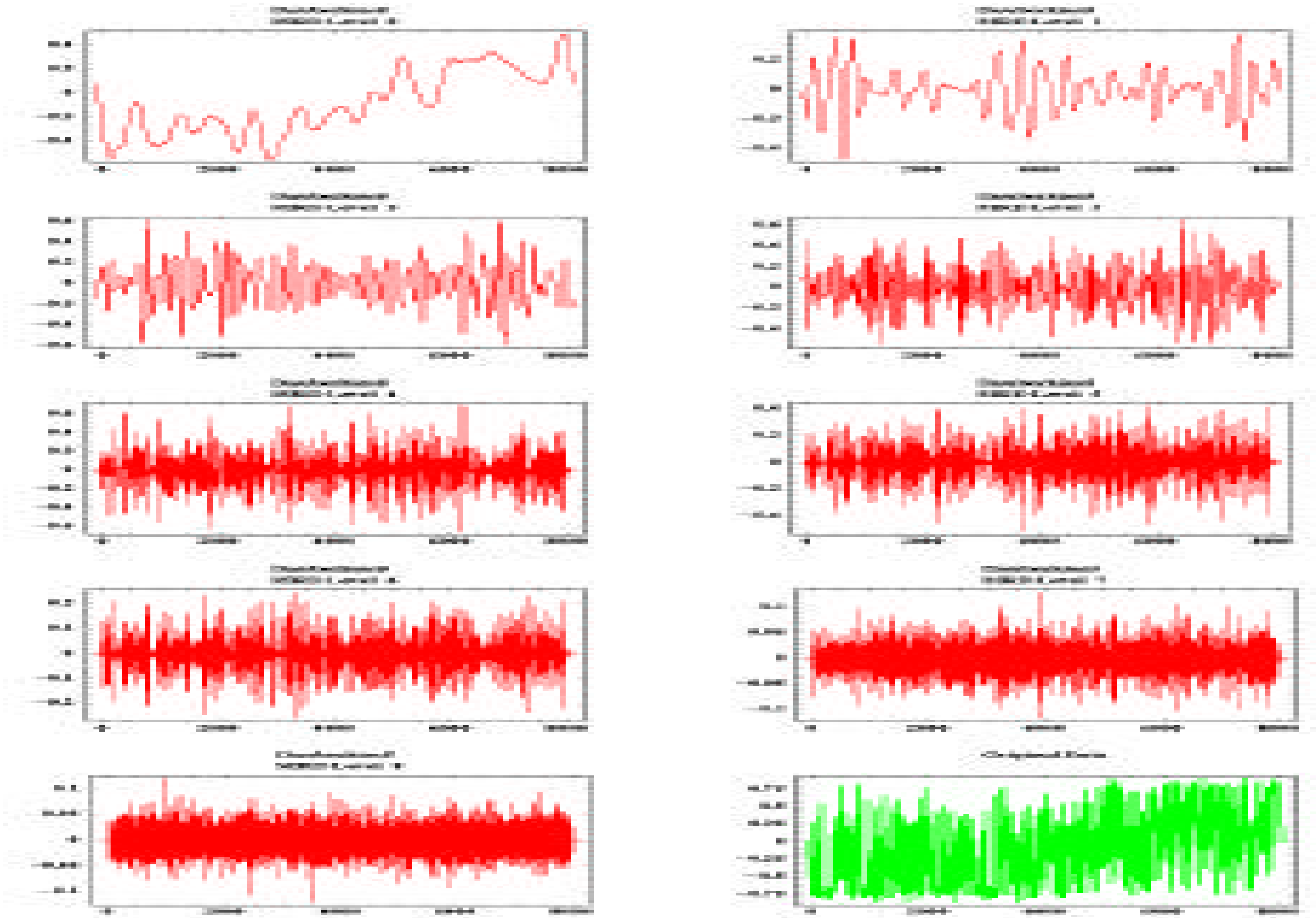
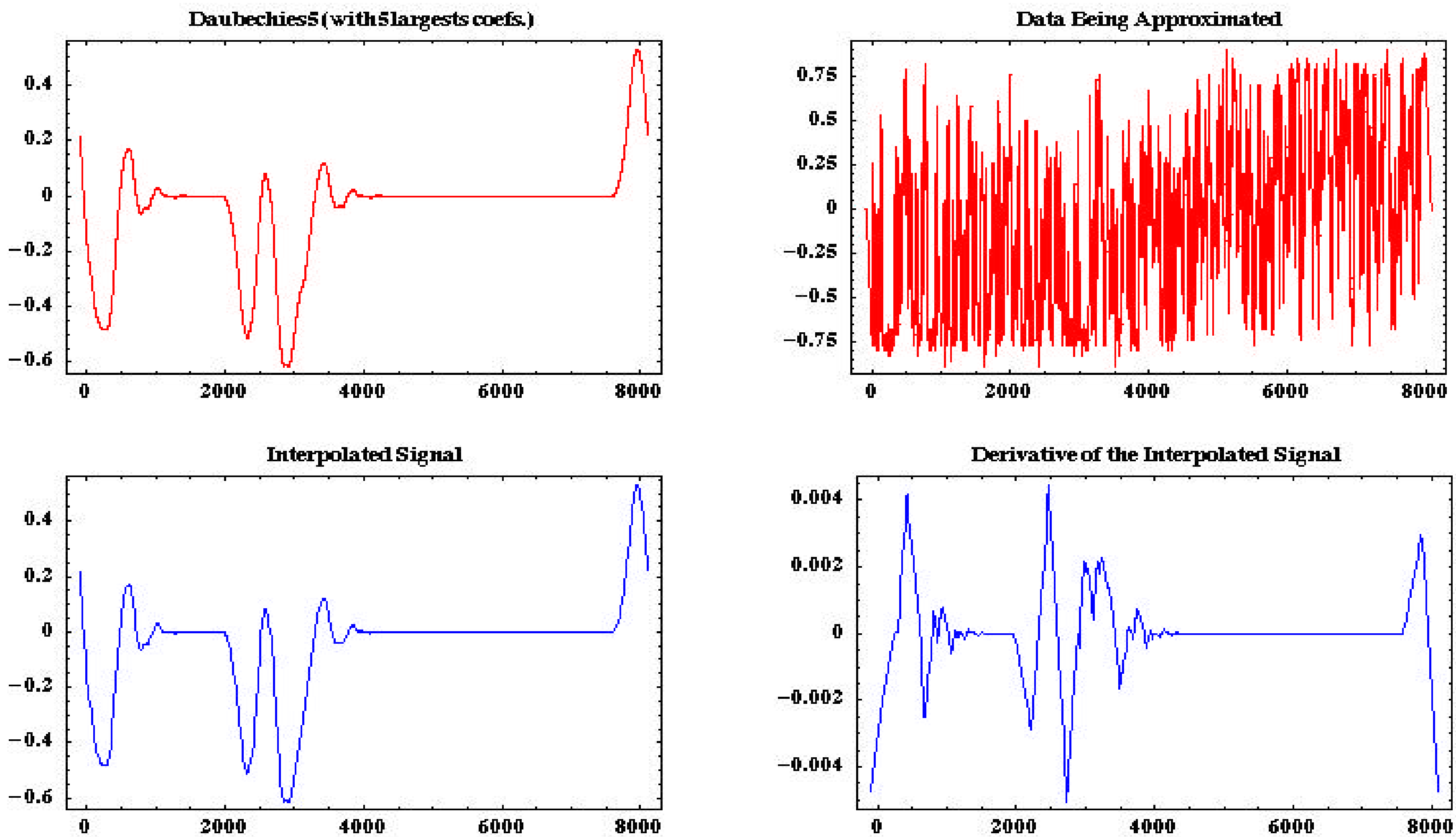
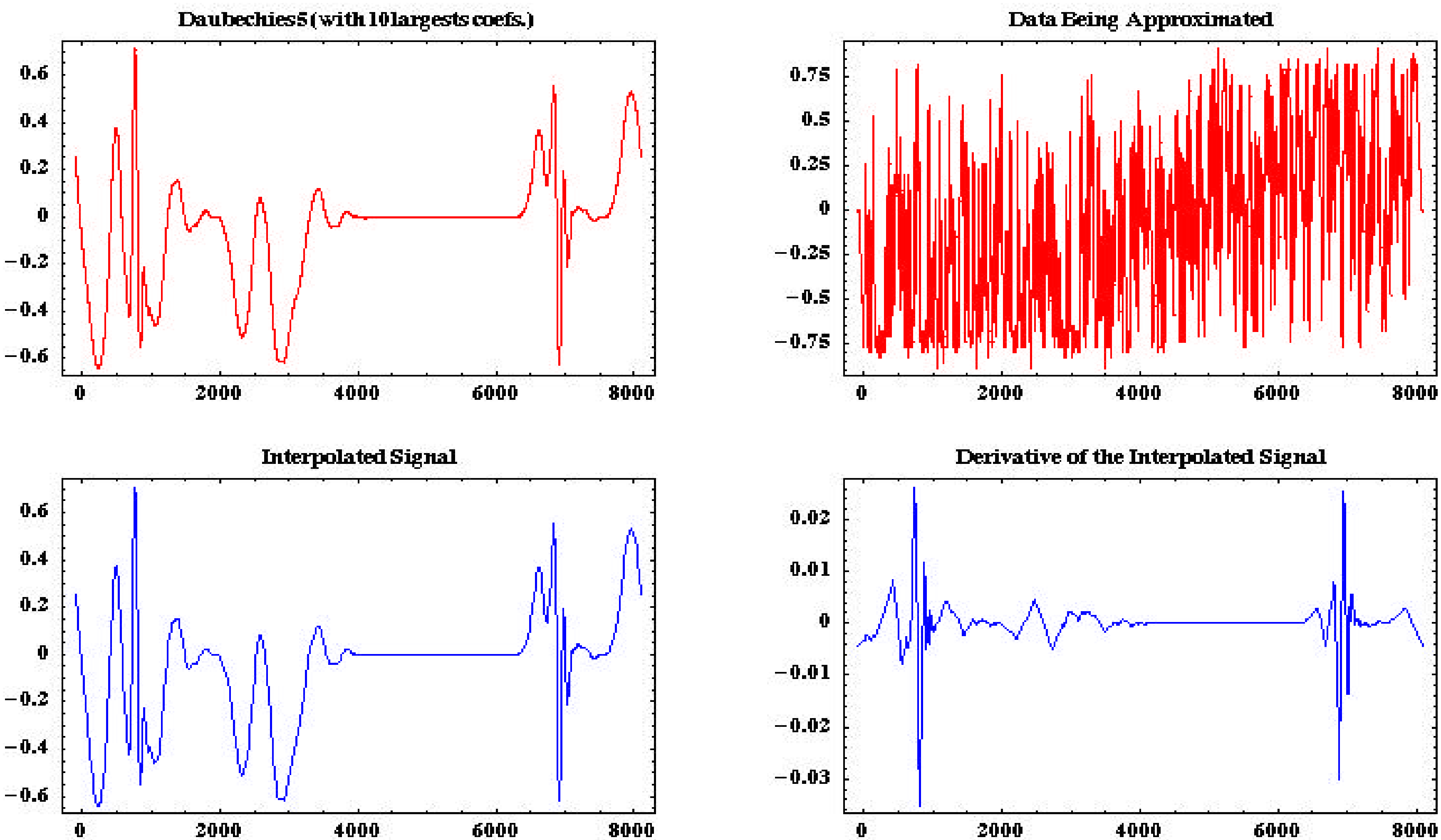


Reconstruction of the Data Using the 5 Largest WLT Coefficients

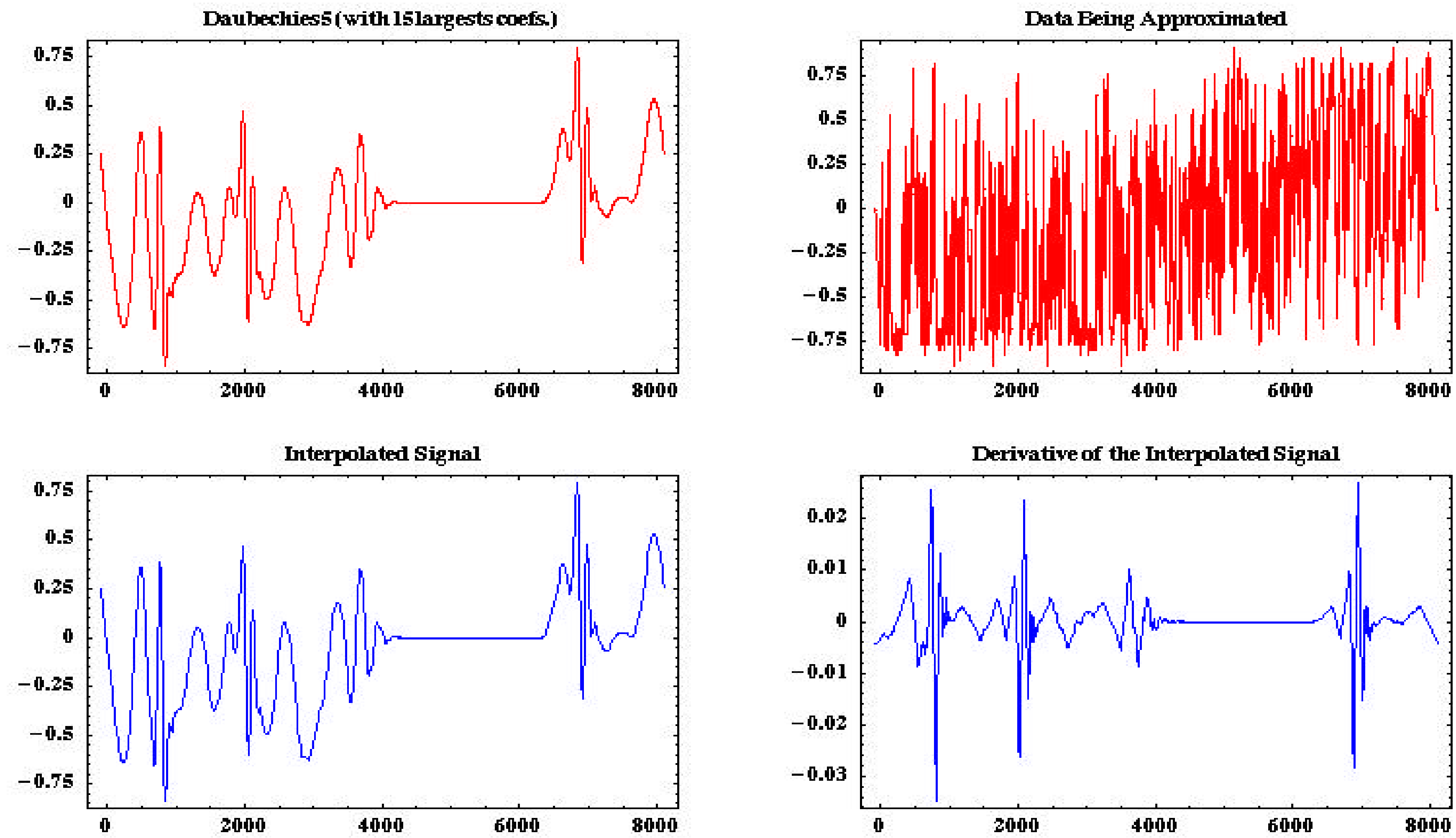


Reconstruction of the Data Using the 10 Largest WLT Coefficients

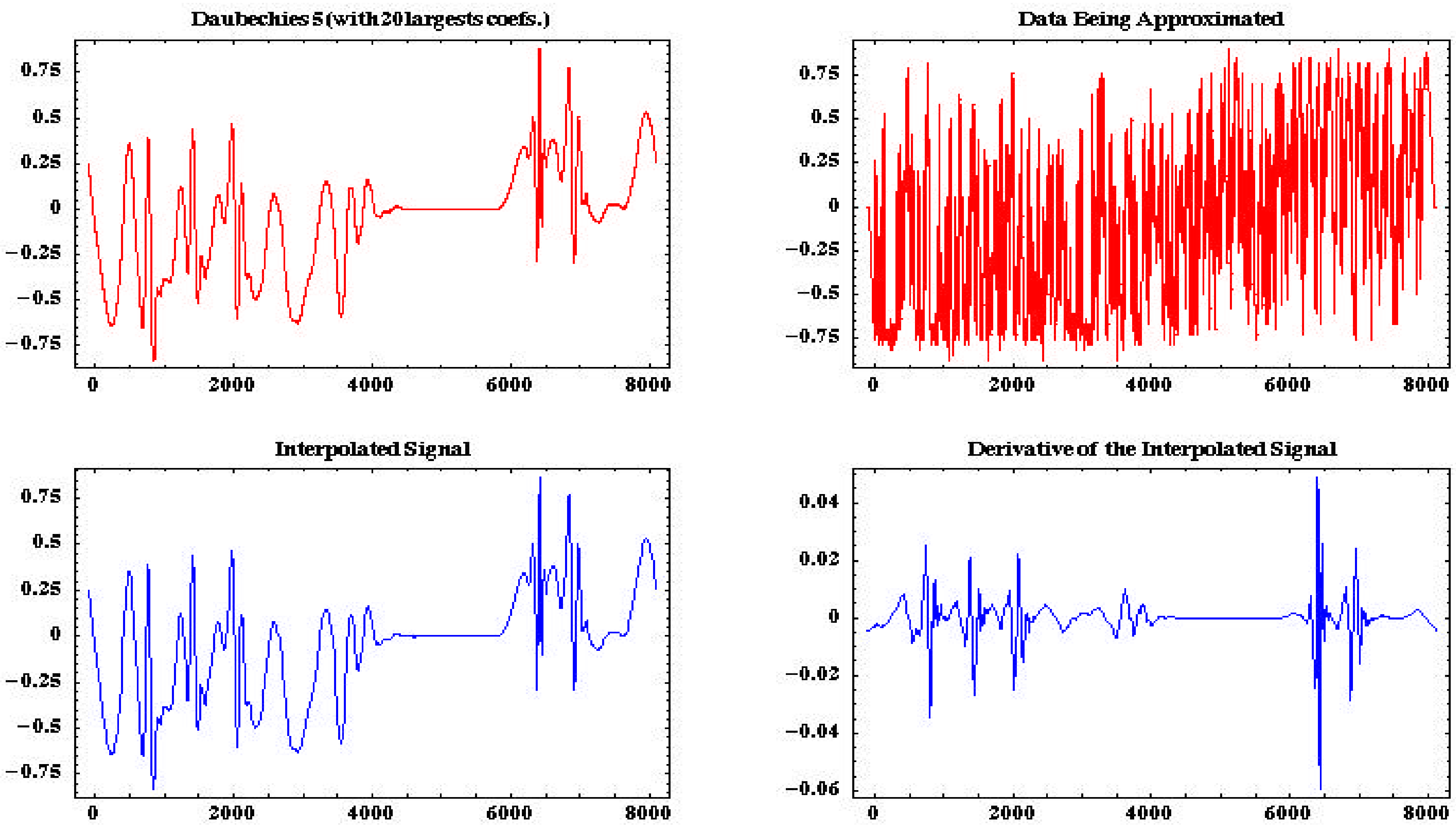
Polymath Research Inc.



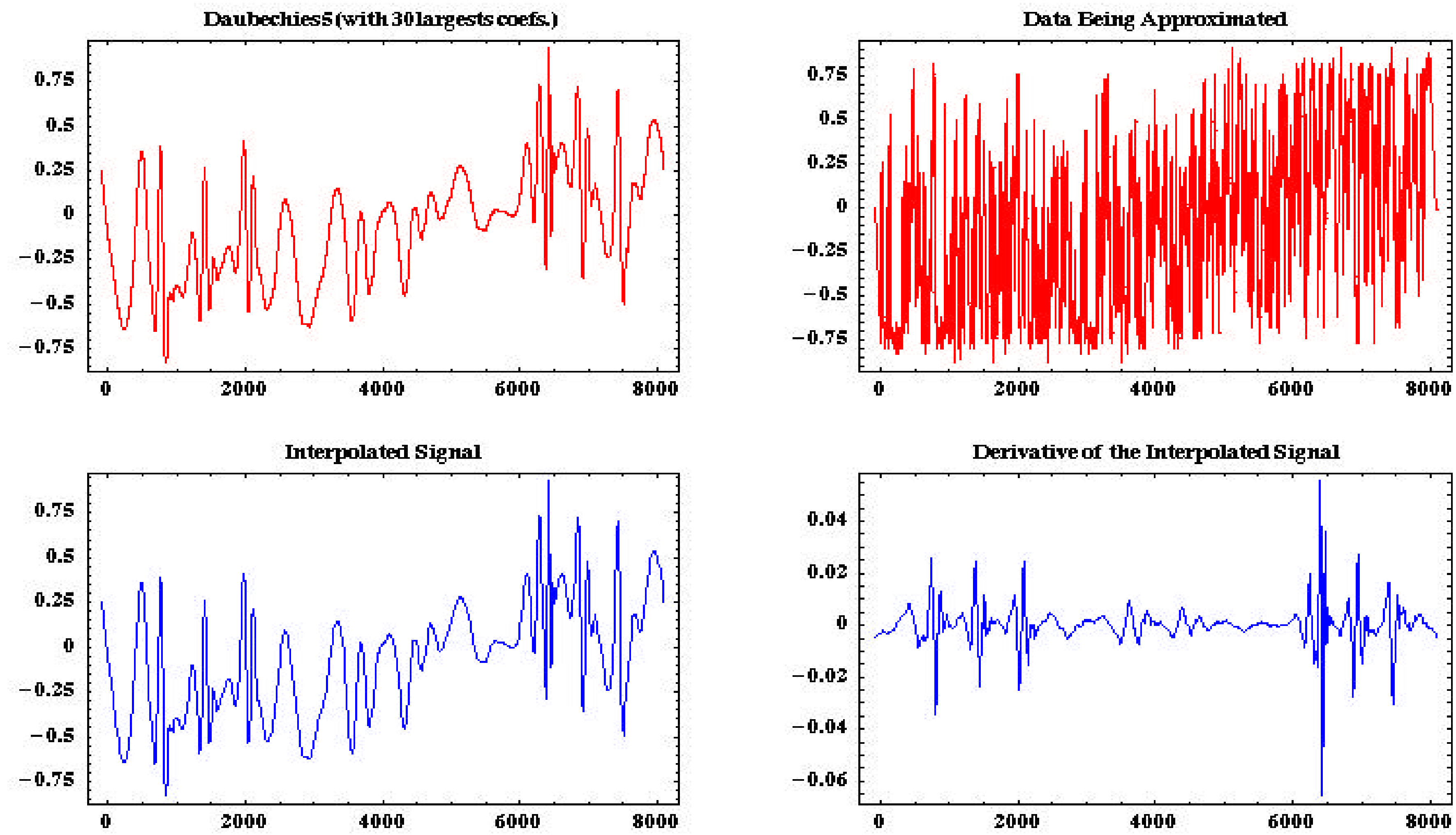
Reconstruction of the Data Using the 15 Largest WLT Coefficients



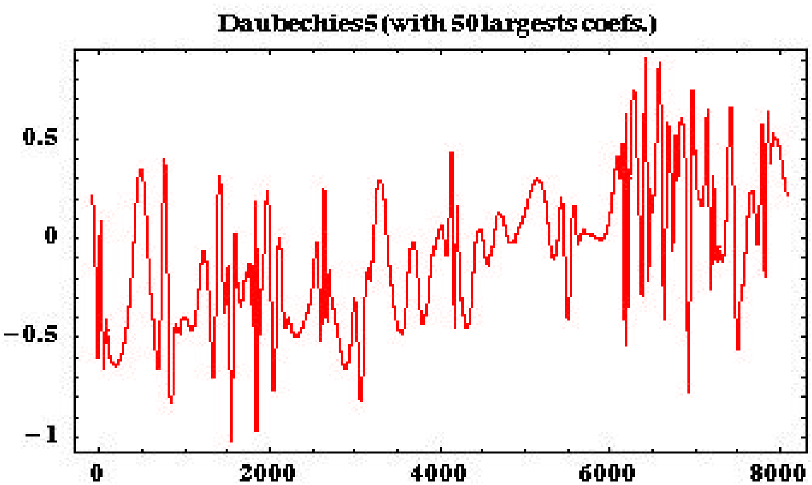
Reconstruction of the Data Using the 20 Largest WLT Coefficients

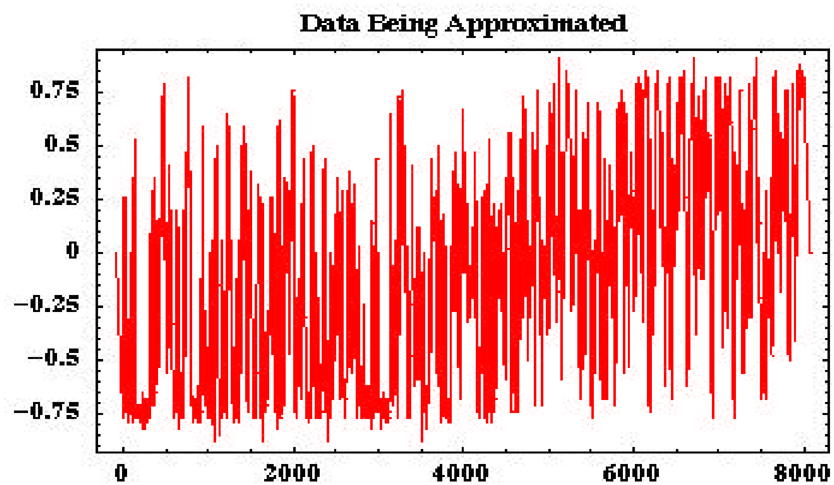


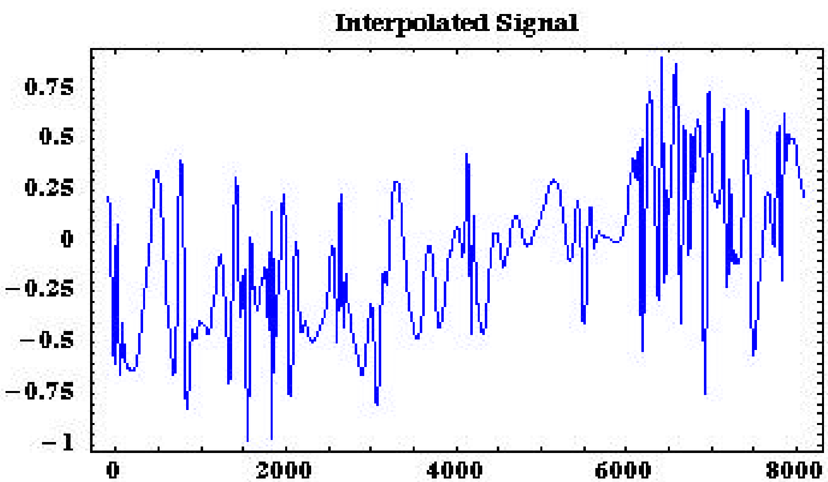
Reconstruction of the Data Using the 30 Largest WLT Coefficients

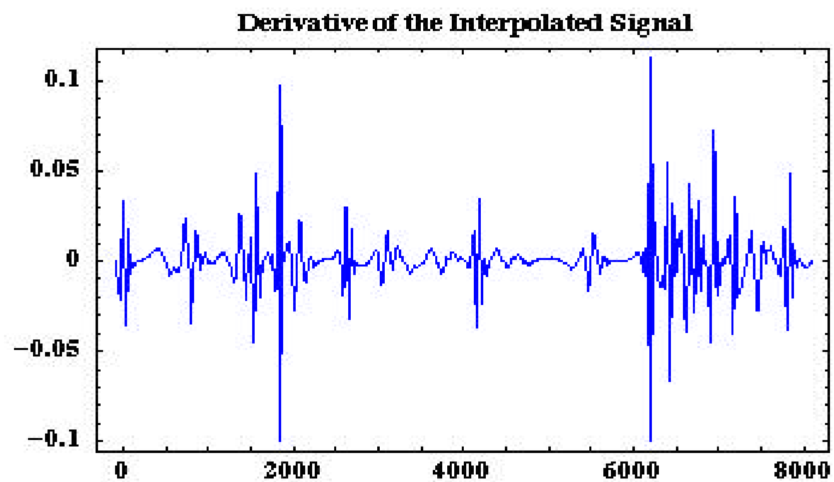


Reconstruction of the Data Using the 50 Largest WLT Coefficients









Conclusions on Raw RT Mix Data Analysis Using DWT

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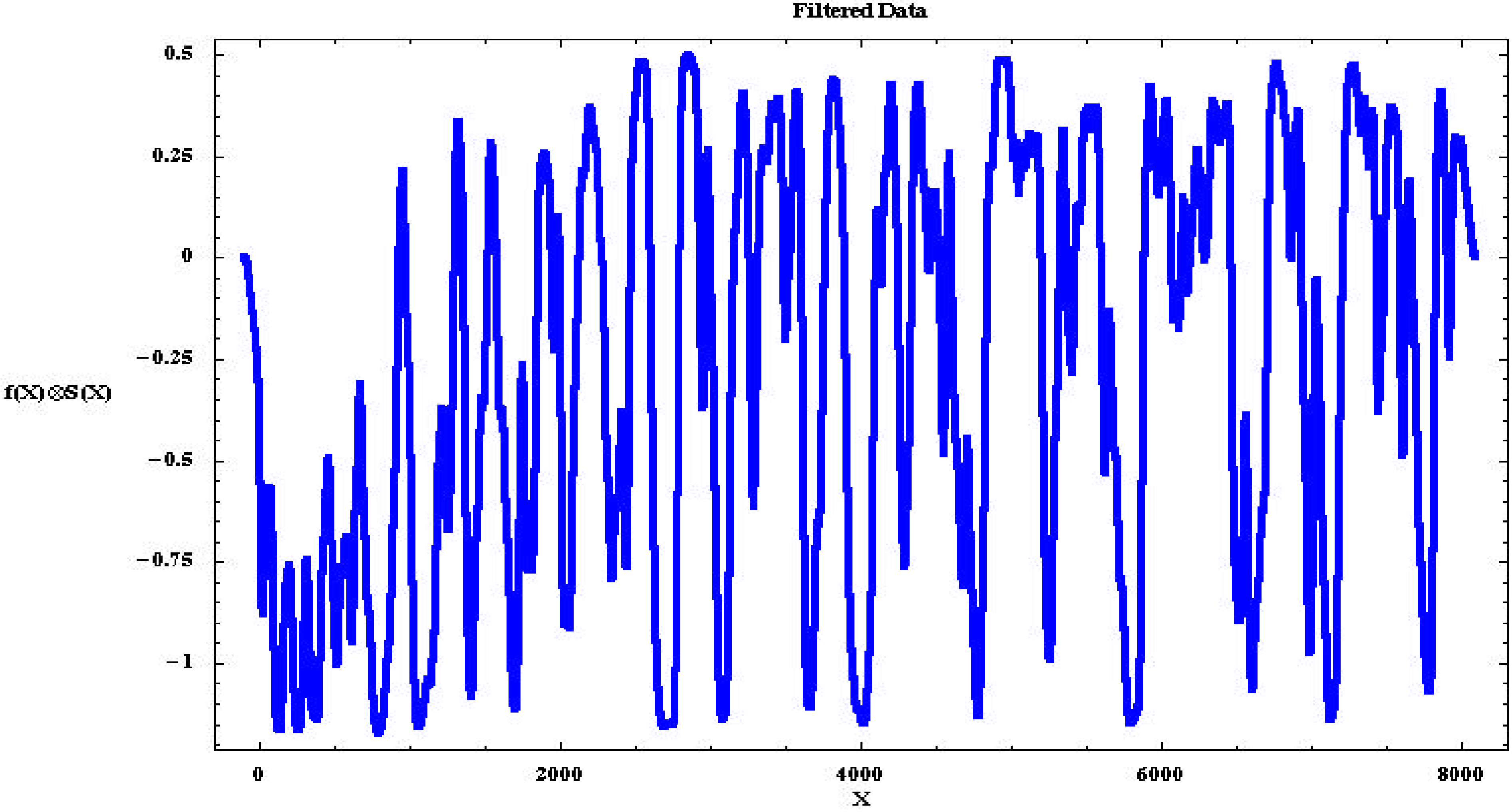
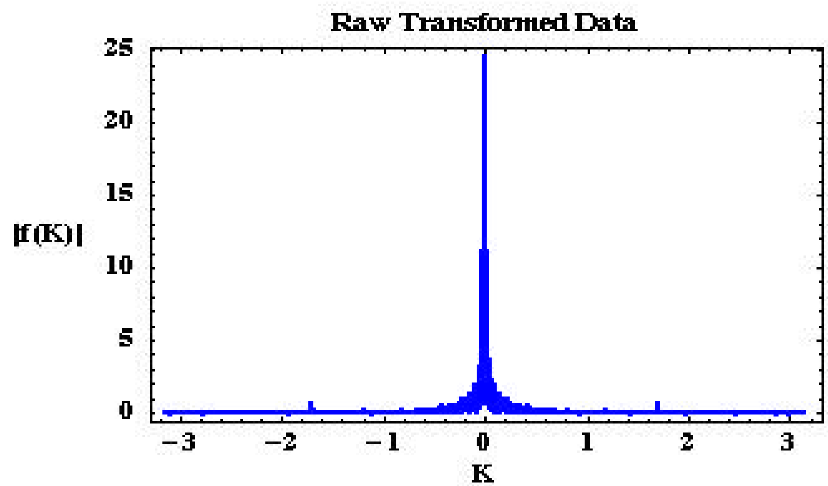
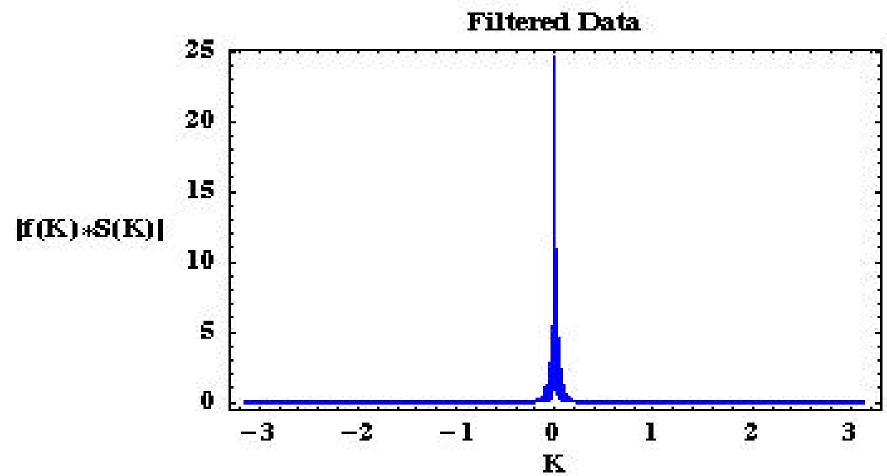
- Compression of around a factor of 20 seems likely with full data set.
- Will see what low pass filtering will do to initial data and its subsequent WLT analysis.
- Looks like 25% of the largest coefficients are enough to reconstruct the clean parts of the data.
- We should compare different stages of evolution of RT Mix in terms of their optimum WLT representations.
- Significant dynamical degrees of freedom vs insignificant ones which vary more slowly or not at all or randomly might be obtainable if we keep at it!

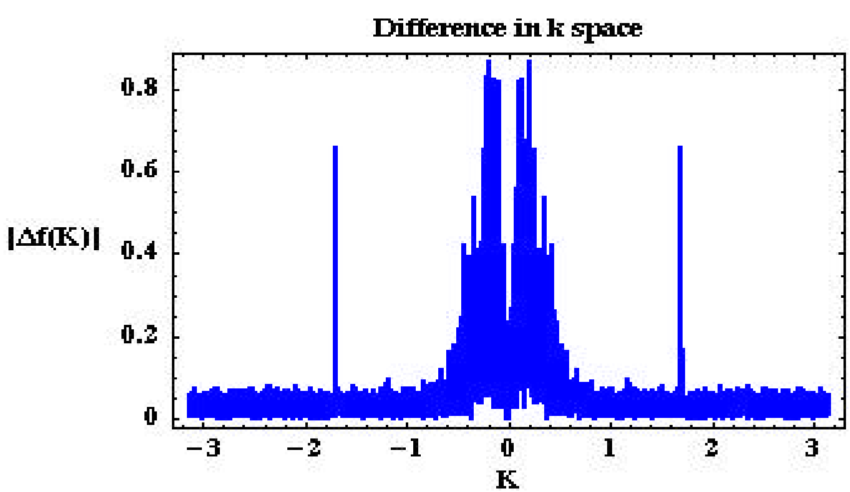
Low Pass Filtered (LPF) Padded and Faded RT Weak Mix Data



The Filtering Has This Form and Effect on the Data in k-Space







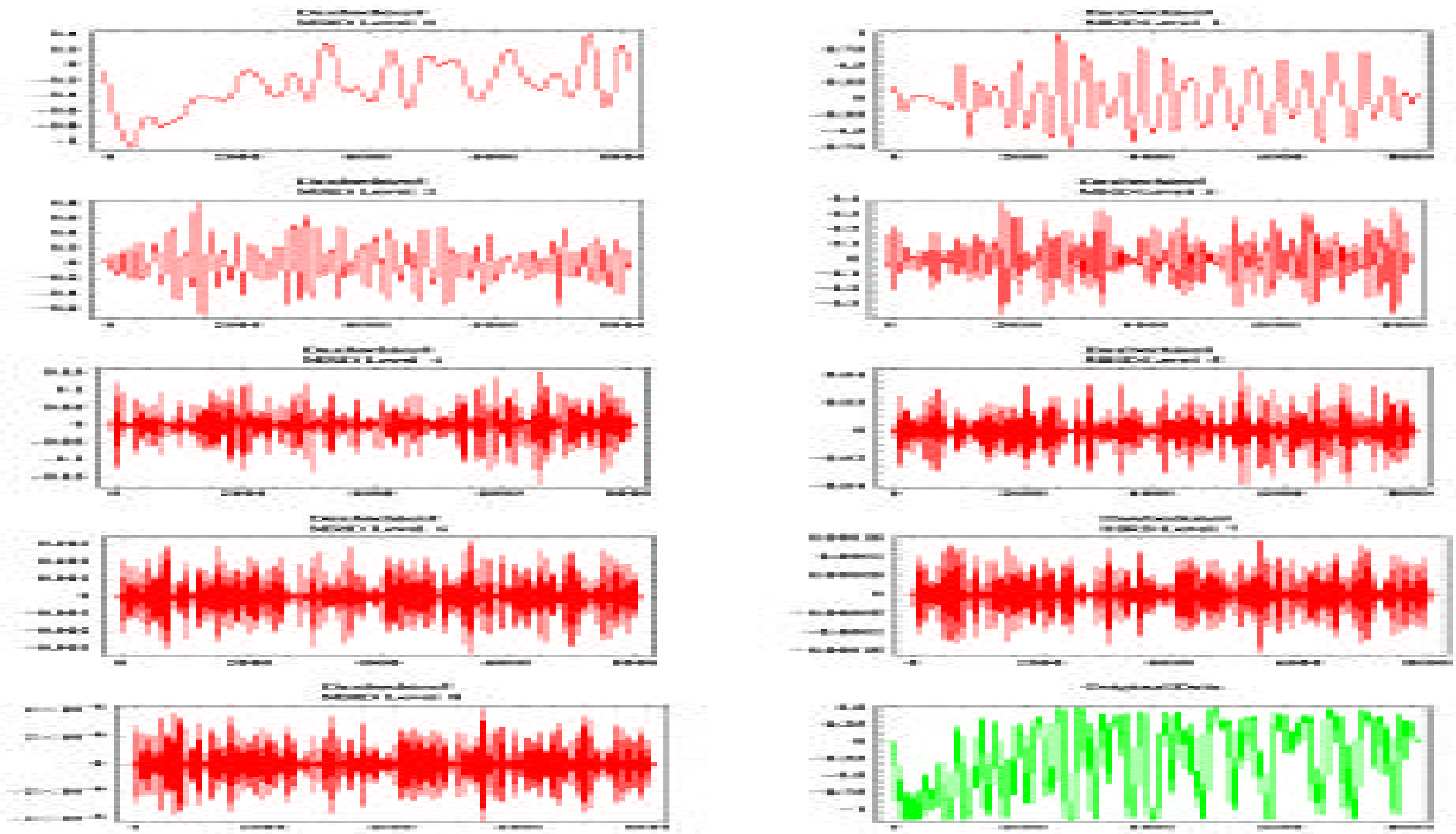
Filter was of the form:

$$S(k) = \exp\left[-\left(\frac{k}{k_{width}}\right)^{2\alpha}\right]$$

Where α=5 and k_{width} = 400

Level by Level Decomposition of the LPF RT Weak Mix Data Using Daub5 WLTs

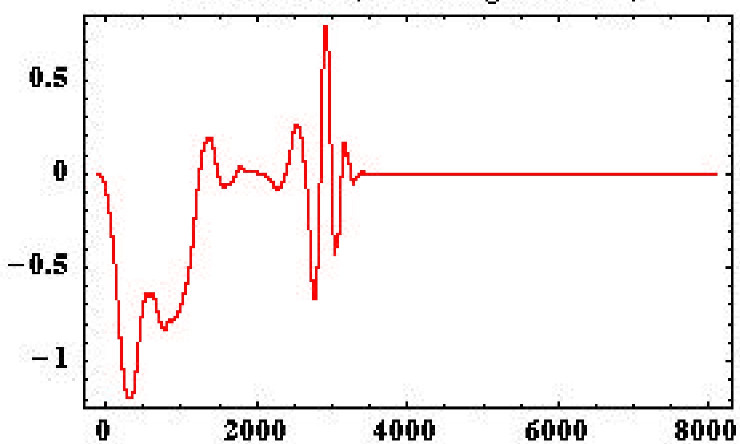
Polymath Research Inc.



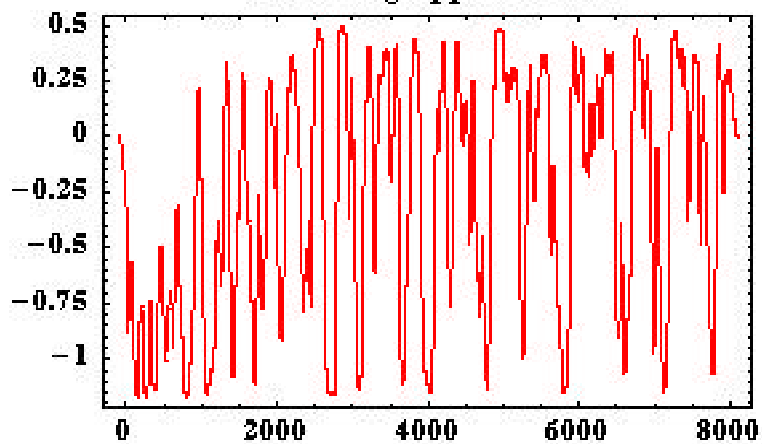
Reconstruction of the LPF RT Weak Mix Data Using the 5 Largest WLT Coefficients

Polymath Research Inc.

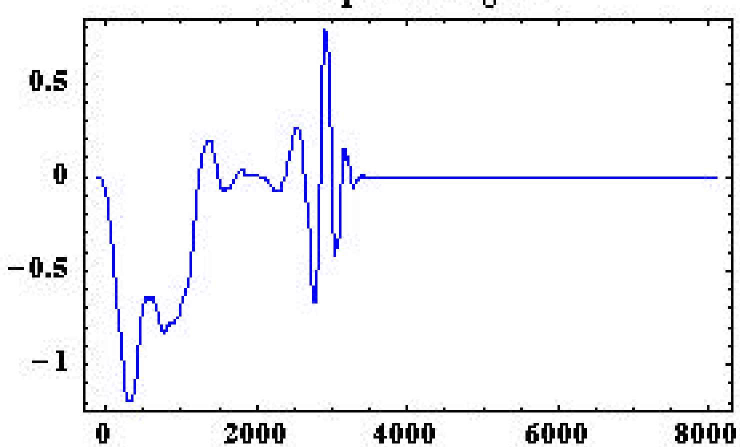
Daubechies5 (with 5 largests coefs.)



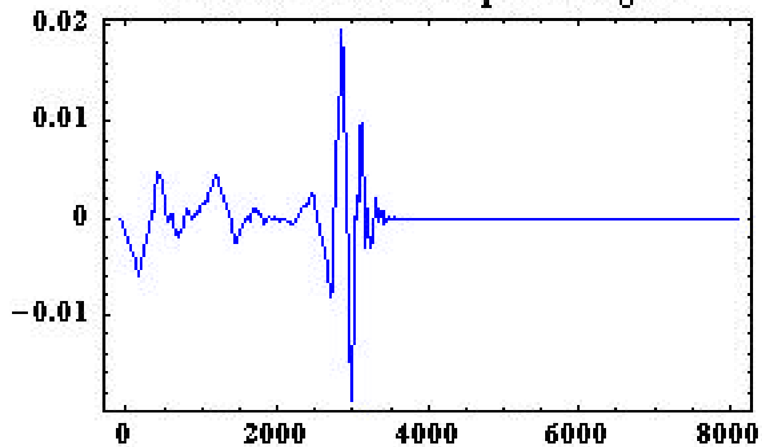
Data Being Approximated



Interpolated Signal

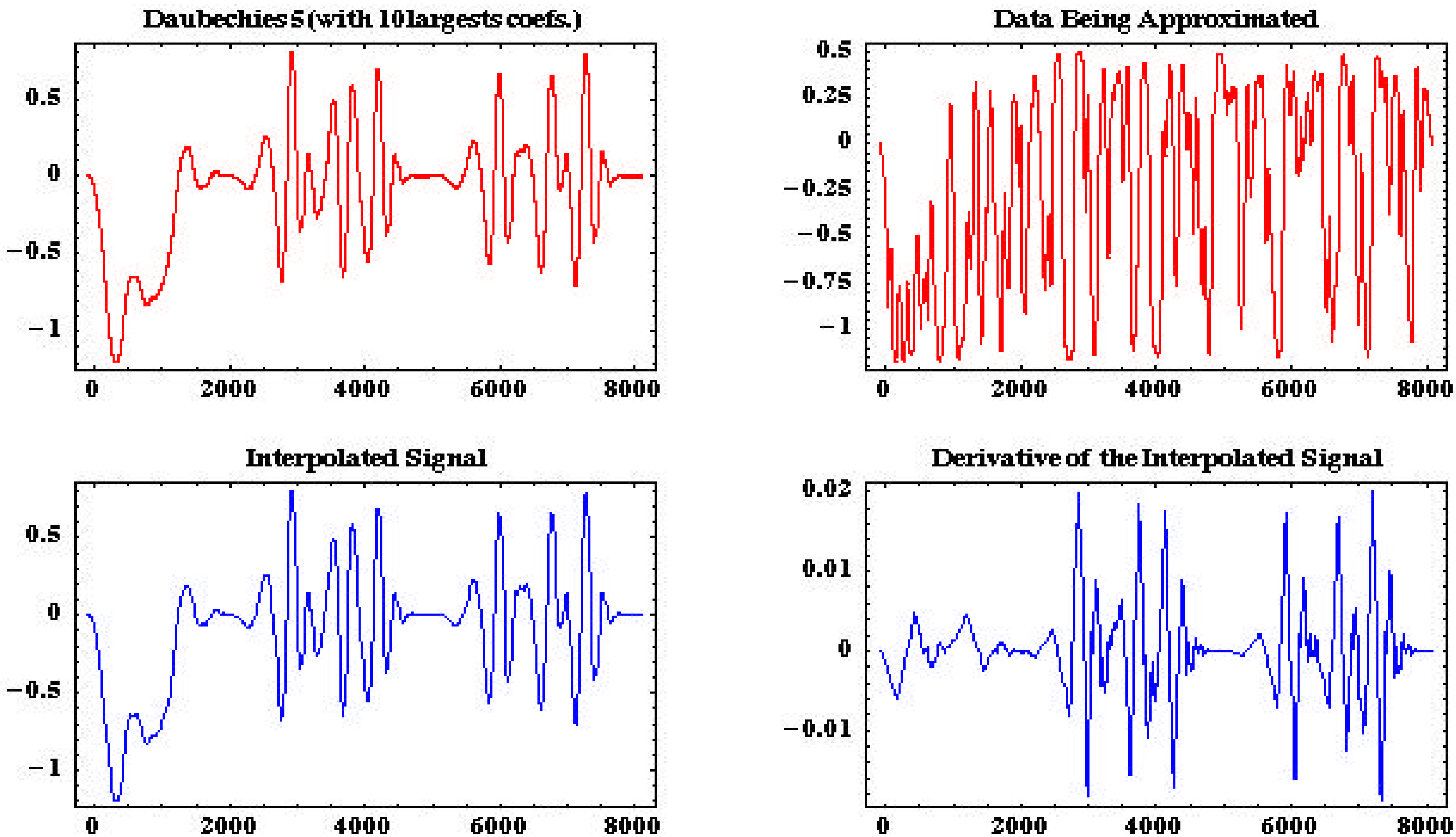


Derivative of the Interpolated Signal



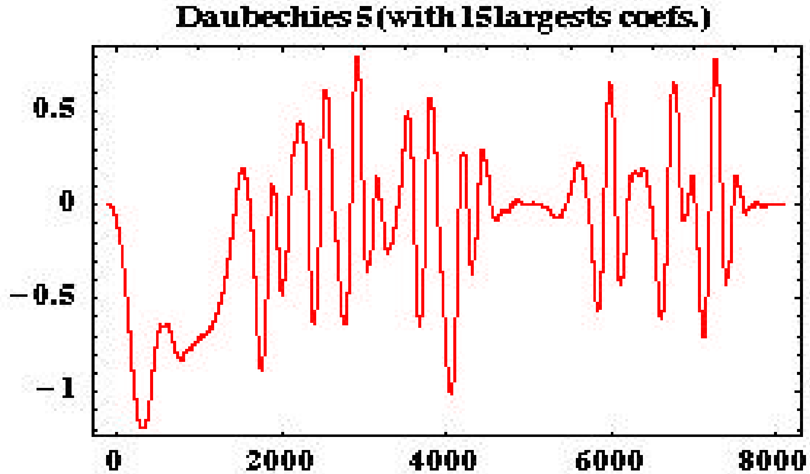
Reconstruction of the LPF RT Weak Mix Data Using the 10 Largest WLT Coefficients

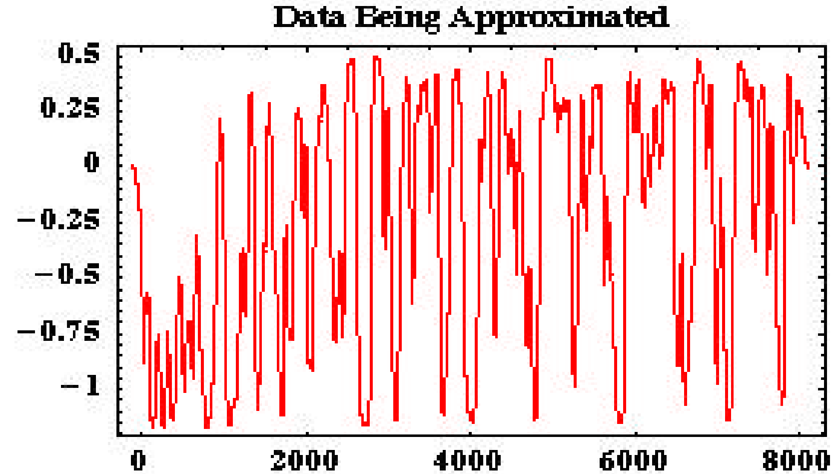
Polymath Research Inc.

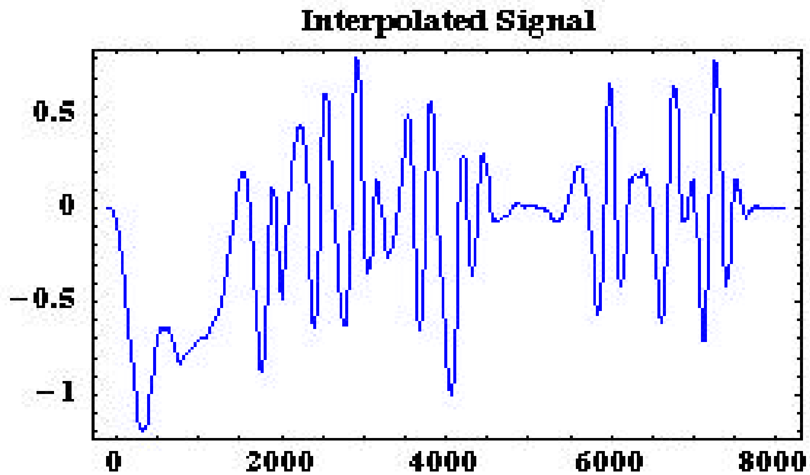


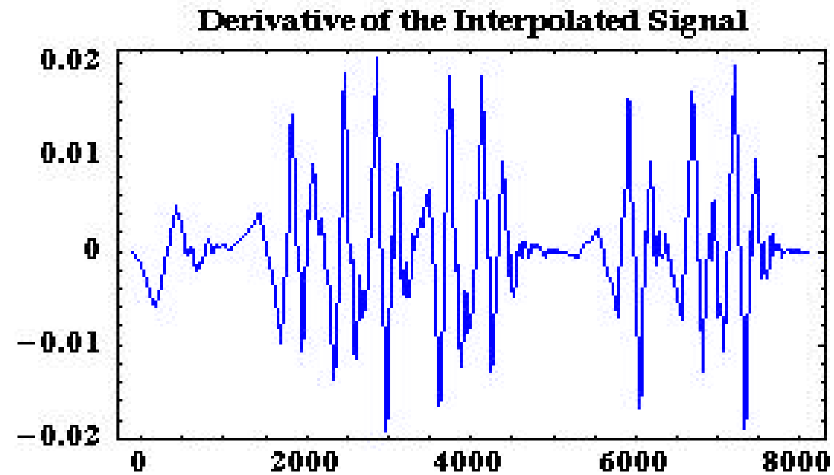
Reconstruction of the LPF RT Weak Mix Data Using the 15 Largest WLT Coefficients

Polymath Research Inc.





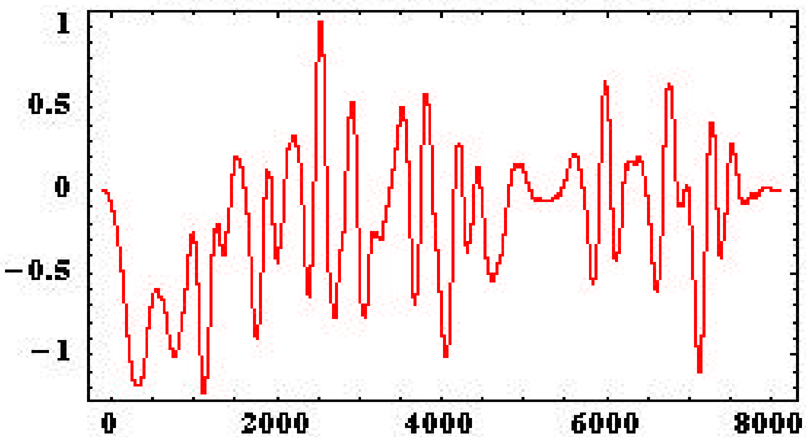




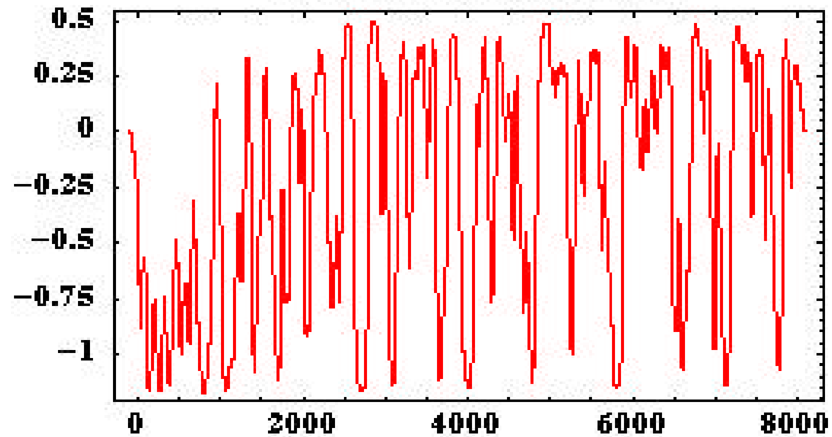
Reconstruction of the LPF RT Weak Mix Data Using the 20 Largest WLT Coefficients

Polymath Research Inc.

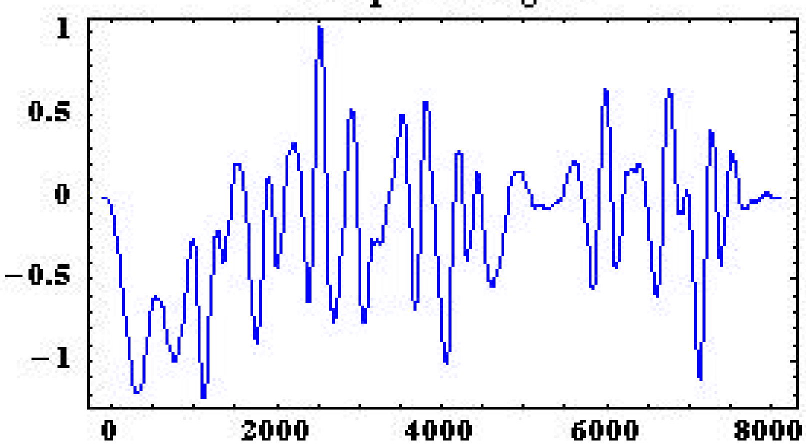
Daubechies 5 (with 20 largests coefs.)



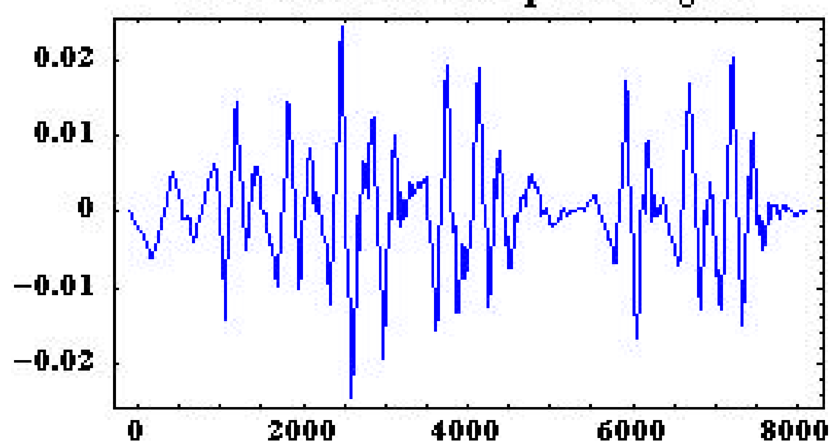
Data Being Approximated



Interpolated Signal

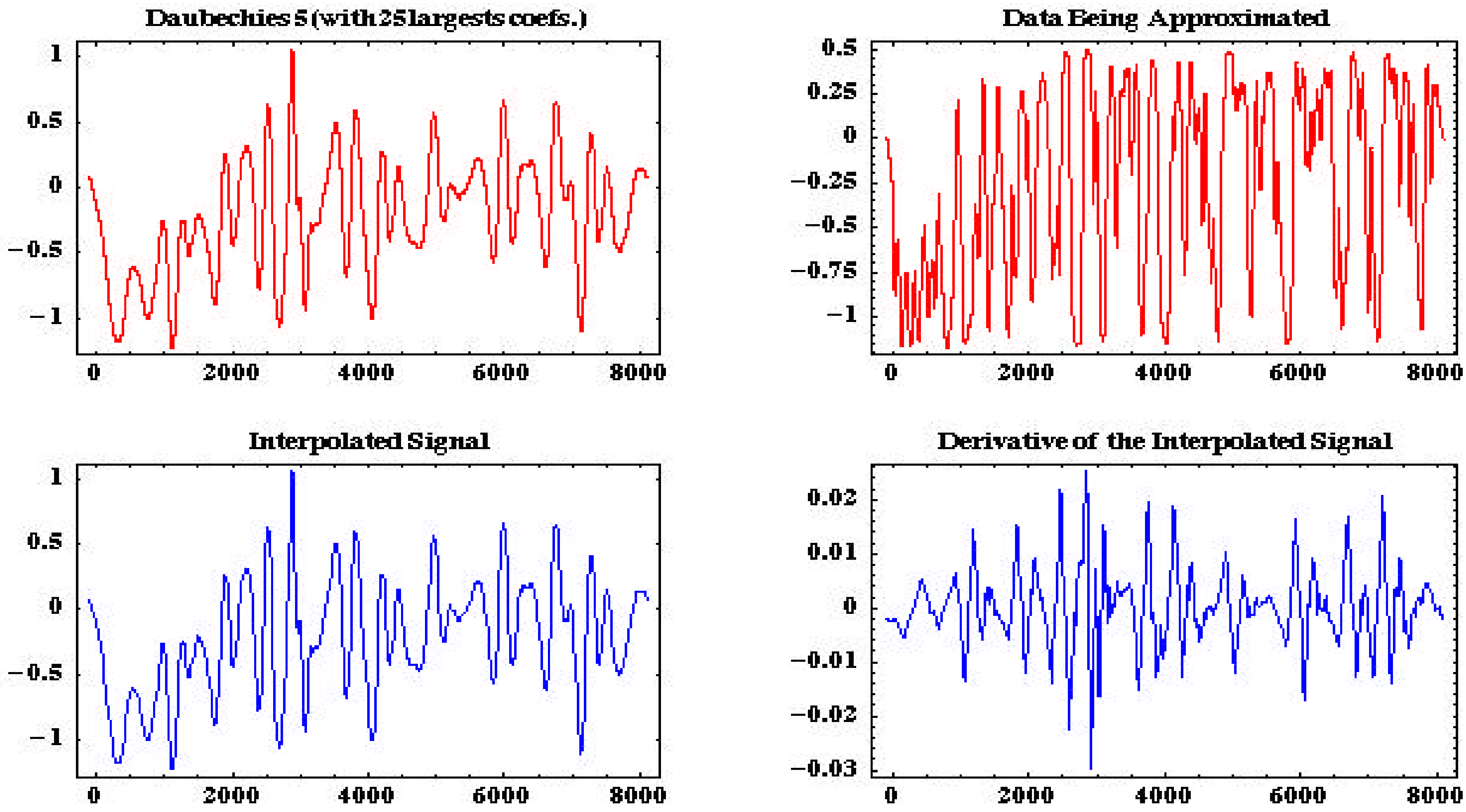


Derivative of the Interpolated Signal



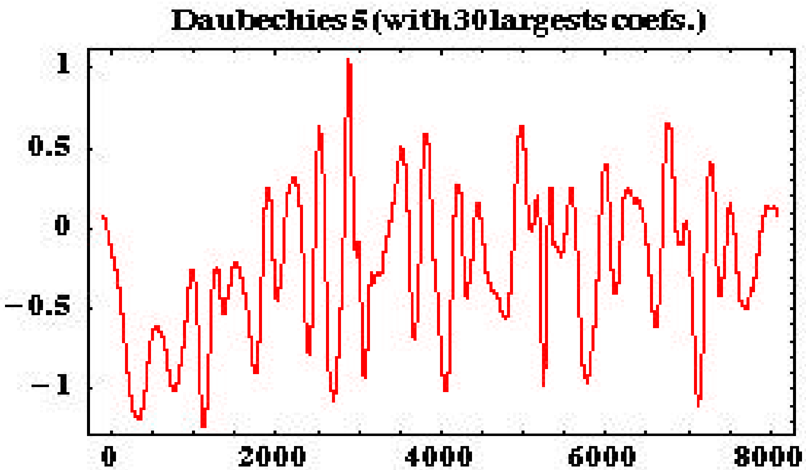
Reconstruction of the LPF RT Weak Mix Data Using the 25 Largest WLT Coefficients

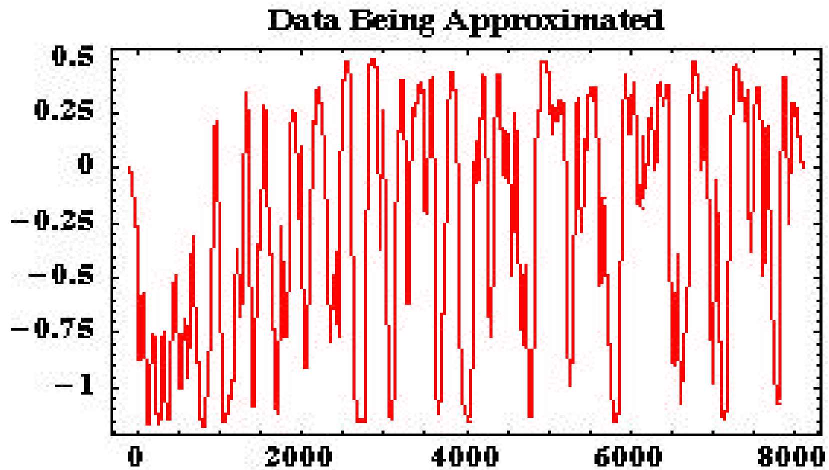
Polymath Research Inc.

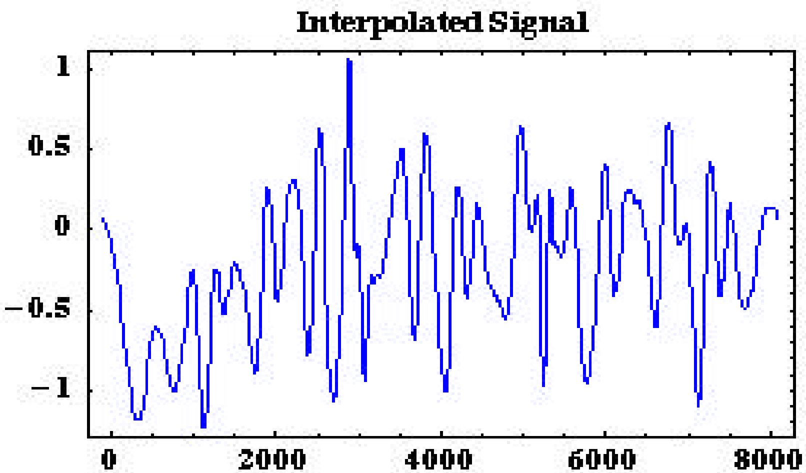


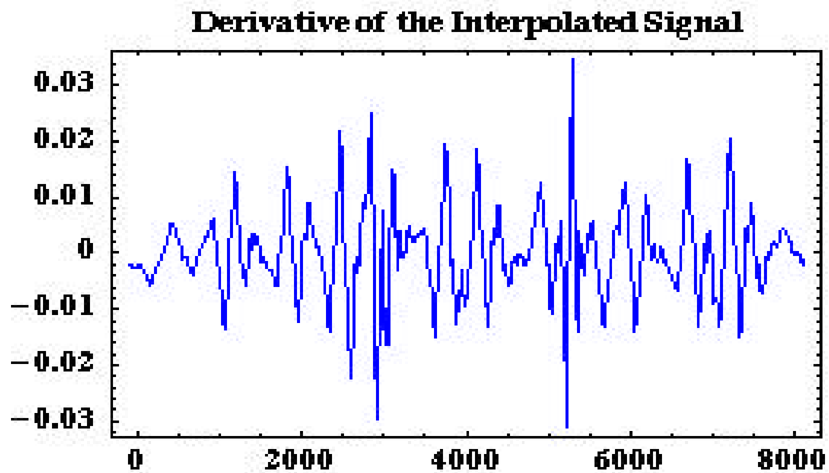
Reconstruction of the LPF RT Weak Mix Data Using the 30 Largest WLT Coefficients

Polymath Research Inc.



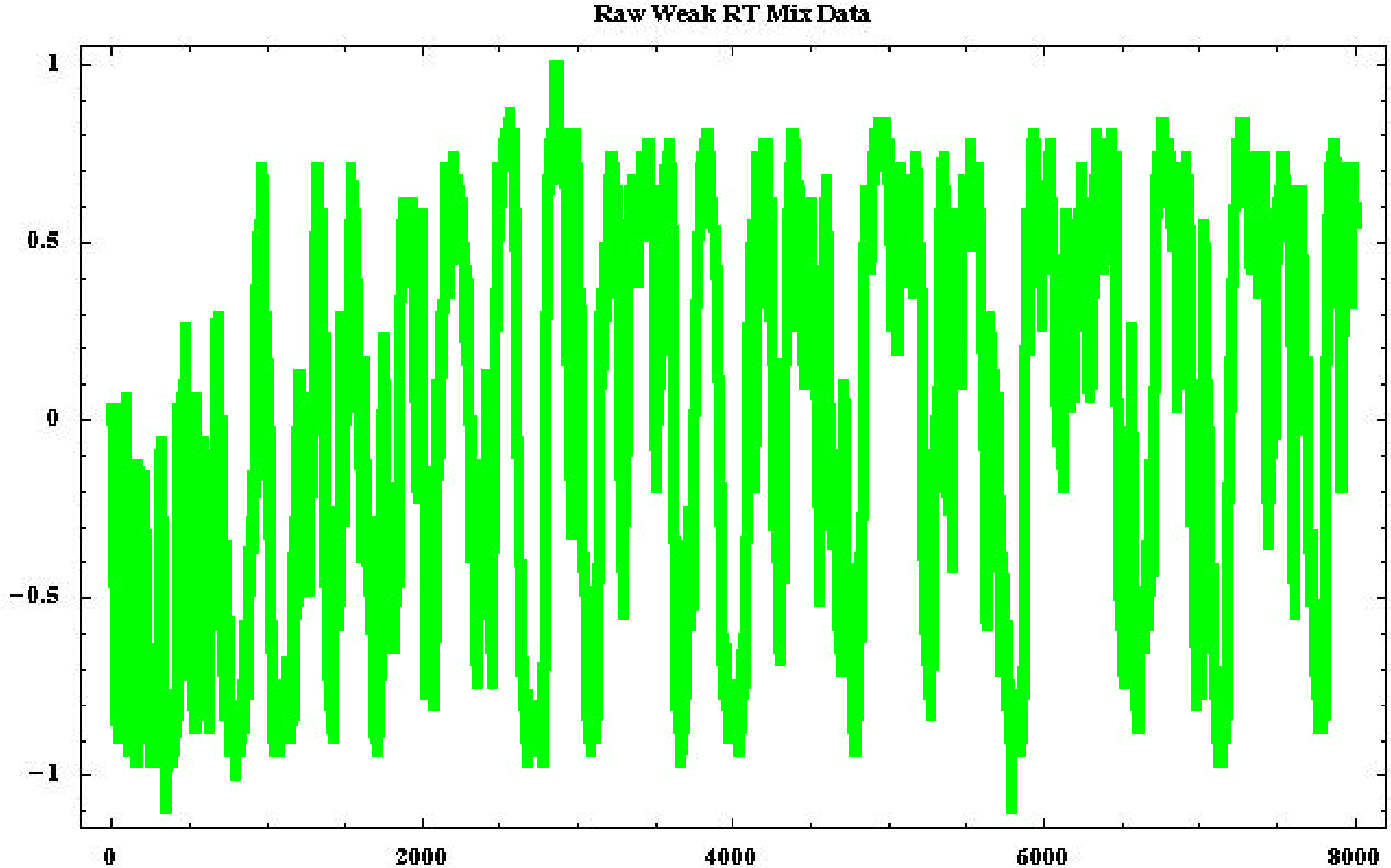






Raw RT Weak Mix Data from Texas A&M (2 cm Downstream, θ = 0.7)

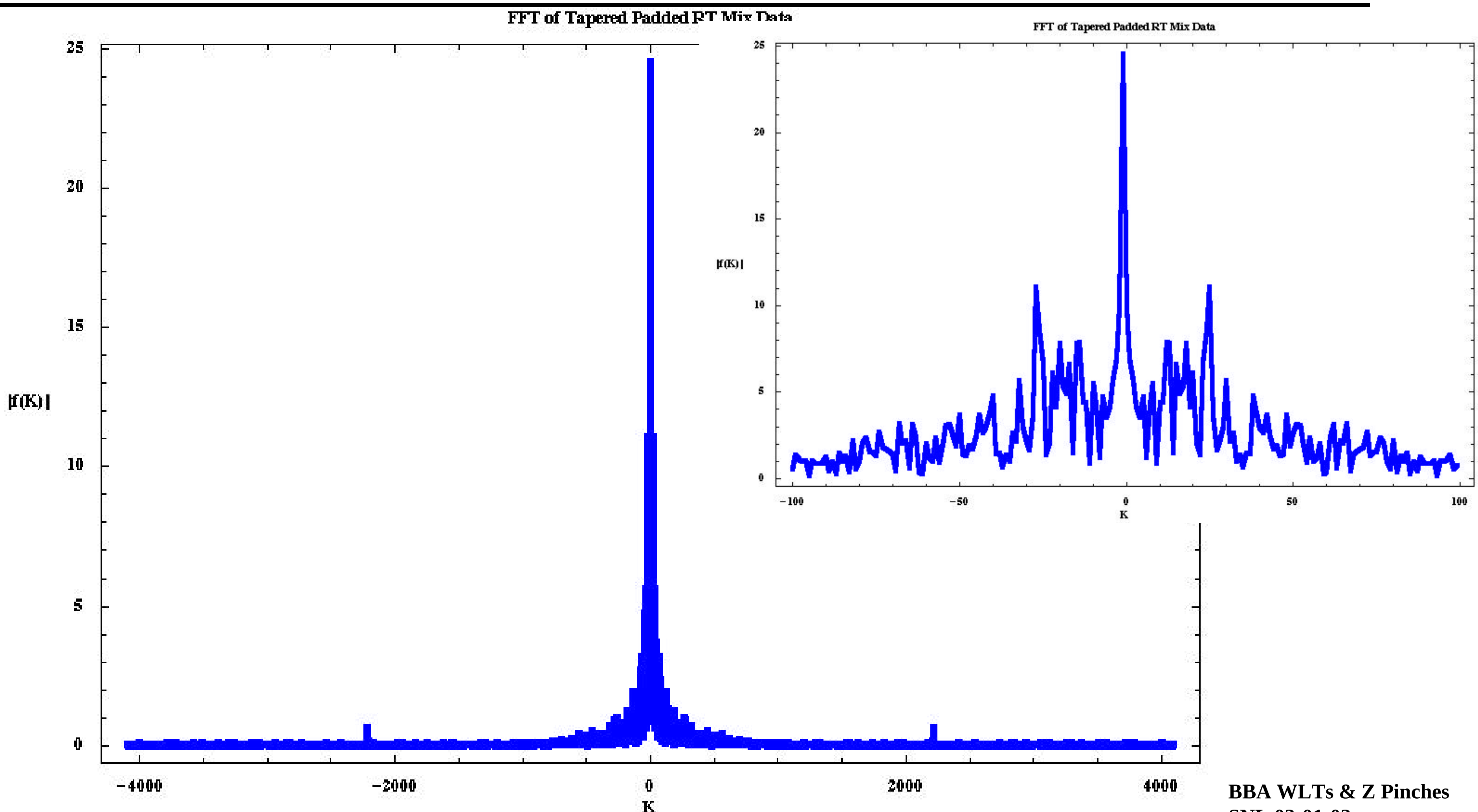
$$\frac{T - T_{AVE}}{T_{MAX} - T_{AVE}}$$



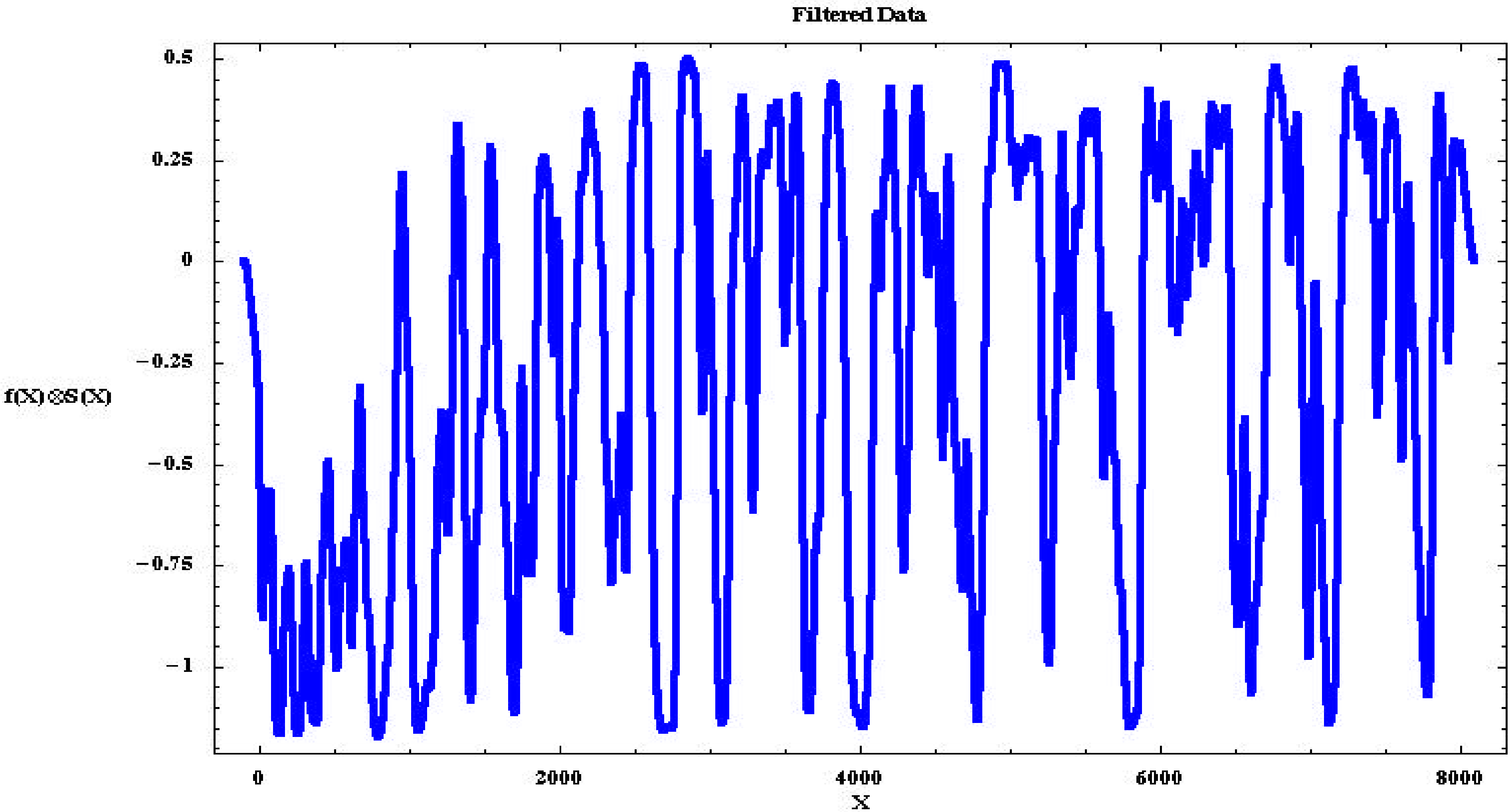
Time, arb. units

The Fourier Transform of the RT Weak Mix Data

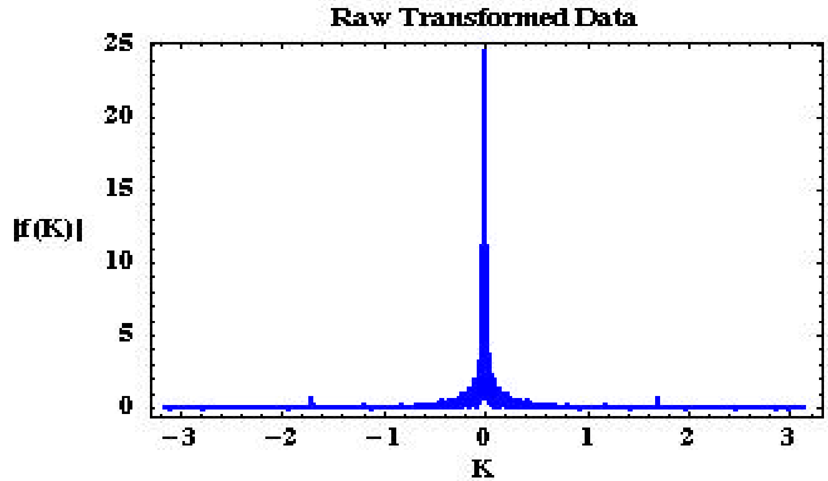
Polymath Research Inc.

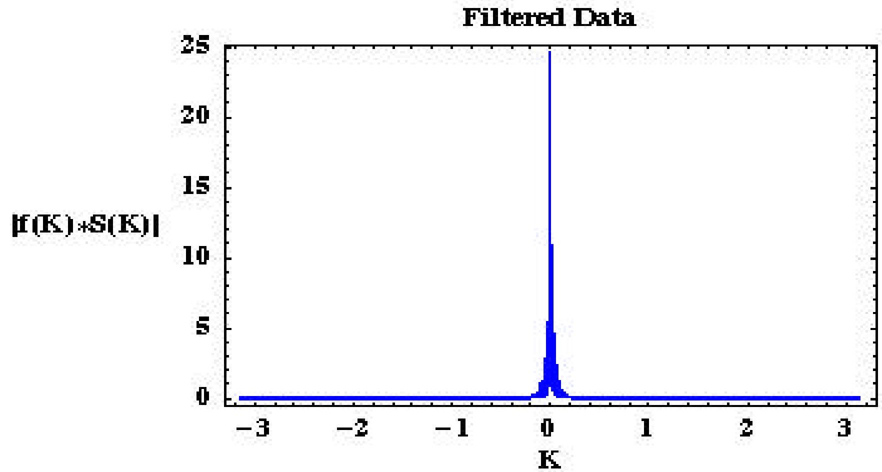


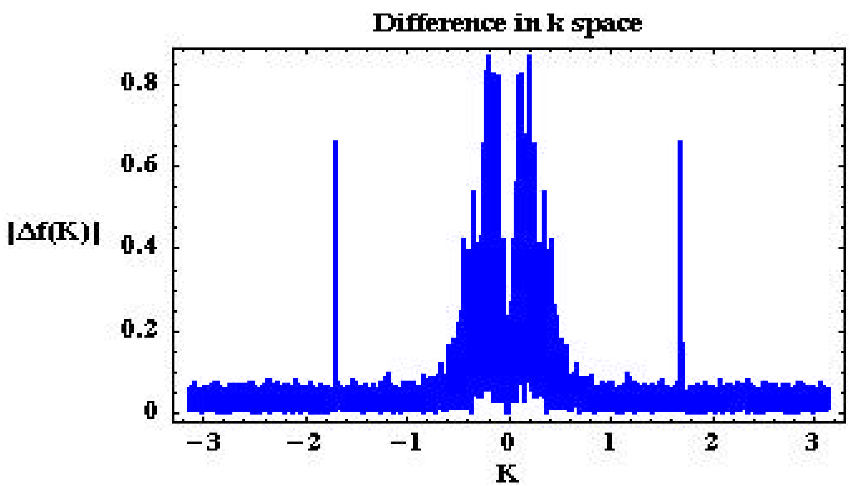
Low Pass Filtered (LPF) Padded and Faded RT Weak Mix Data



The Filtering Has This Form and Effect on the Data in k-Space





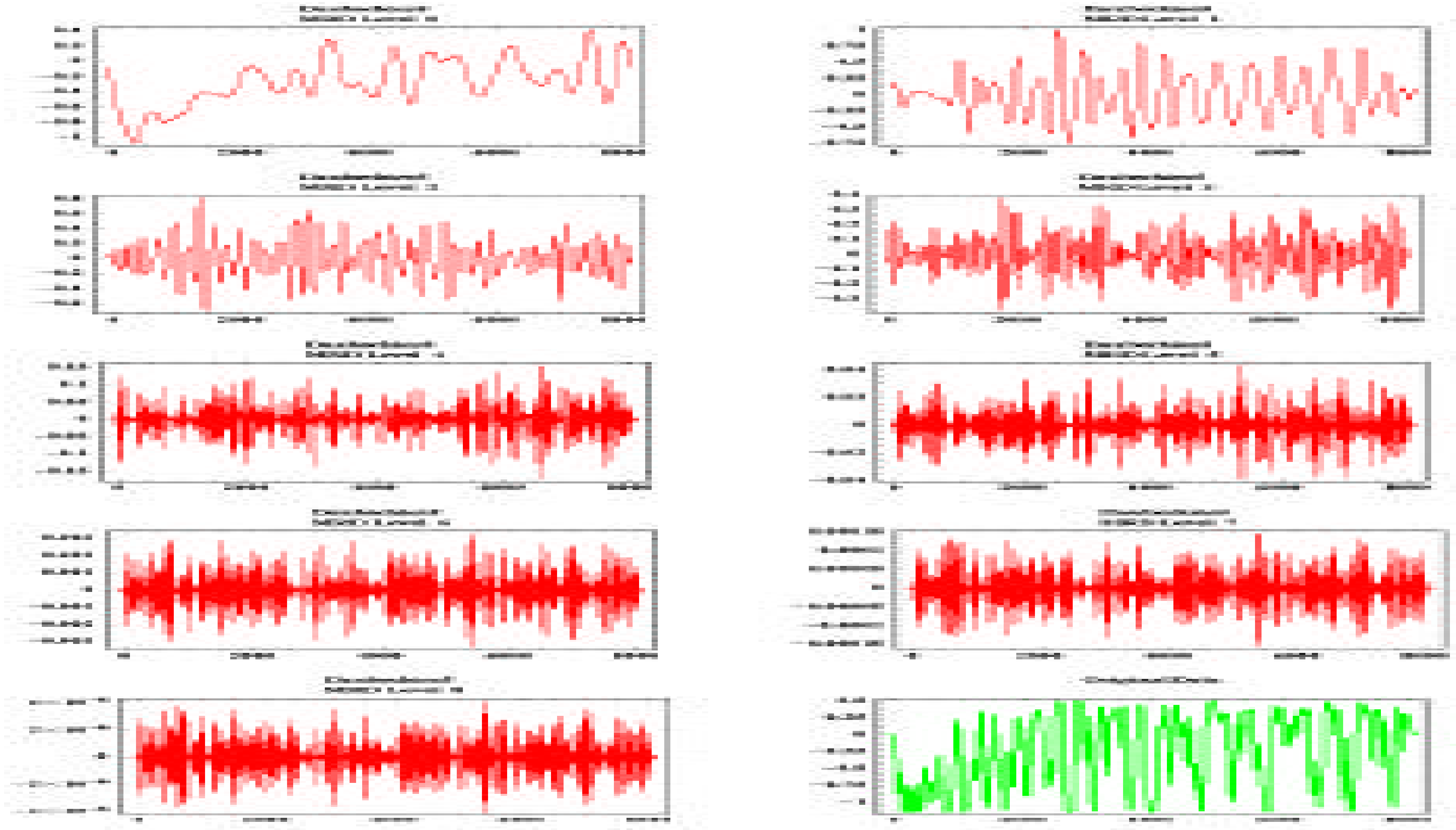


Filter was of the form:

$$S(k) = \exp\left[-\left(\frac{k}{k_{width}}\right)^{2\alpha}\right]$$

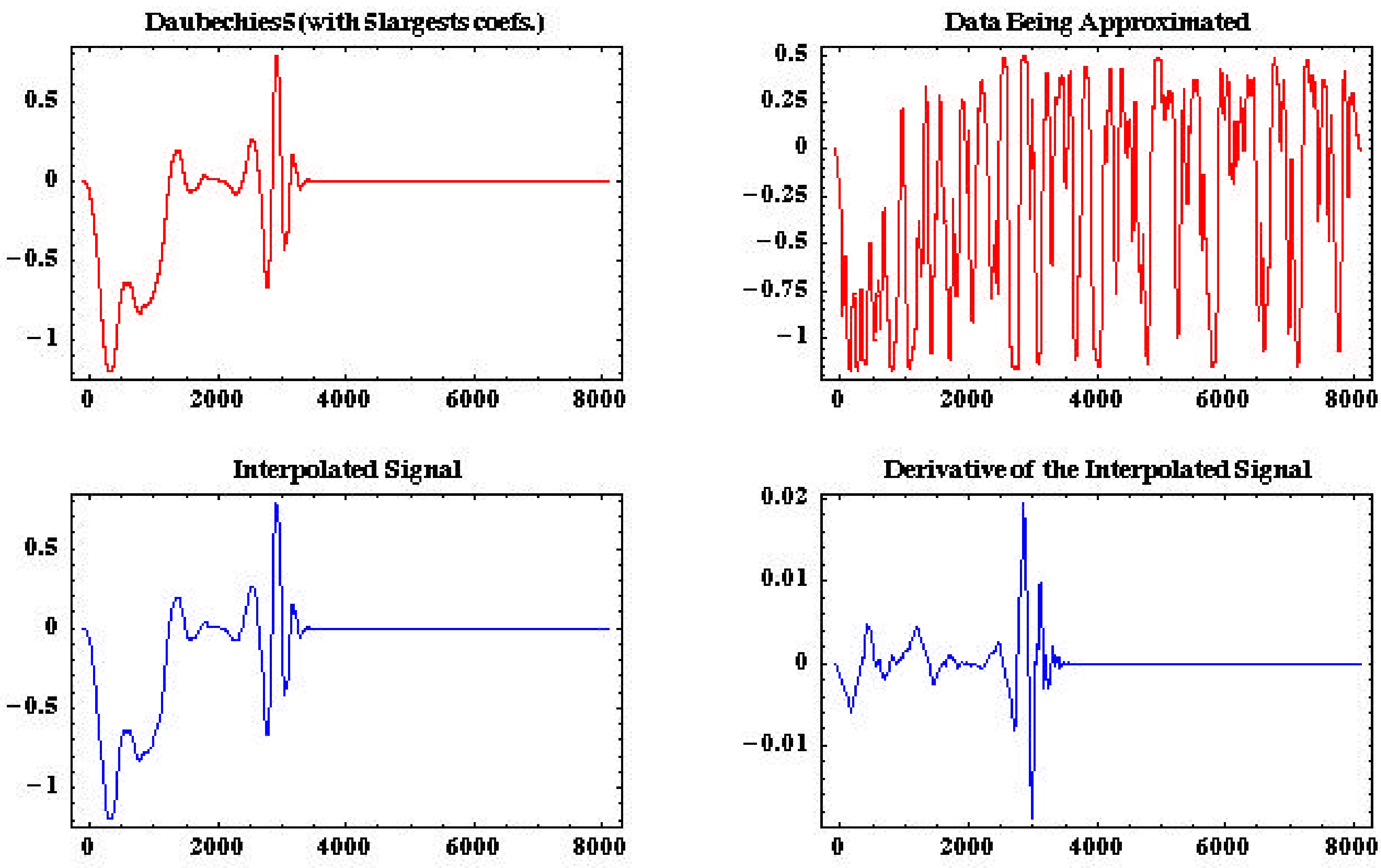
Where α=5 and k_{width} = 400

Level by Level Decomposition of the LPF RT Weak Mix Data Using Daub5 WLTs



Reconstruction of the LPF RT Weak Mix Data Using the 5 Largest WLT Coefficients

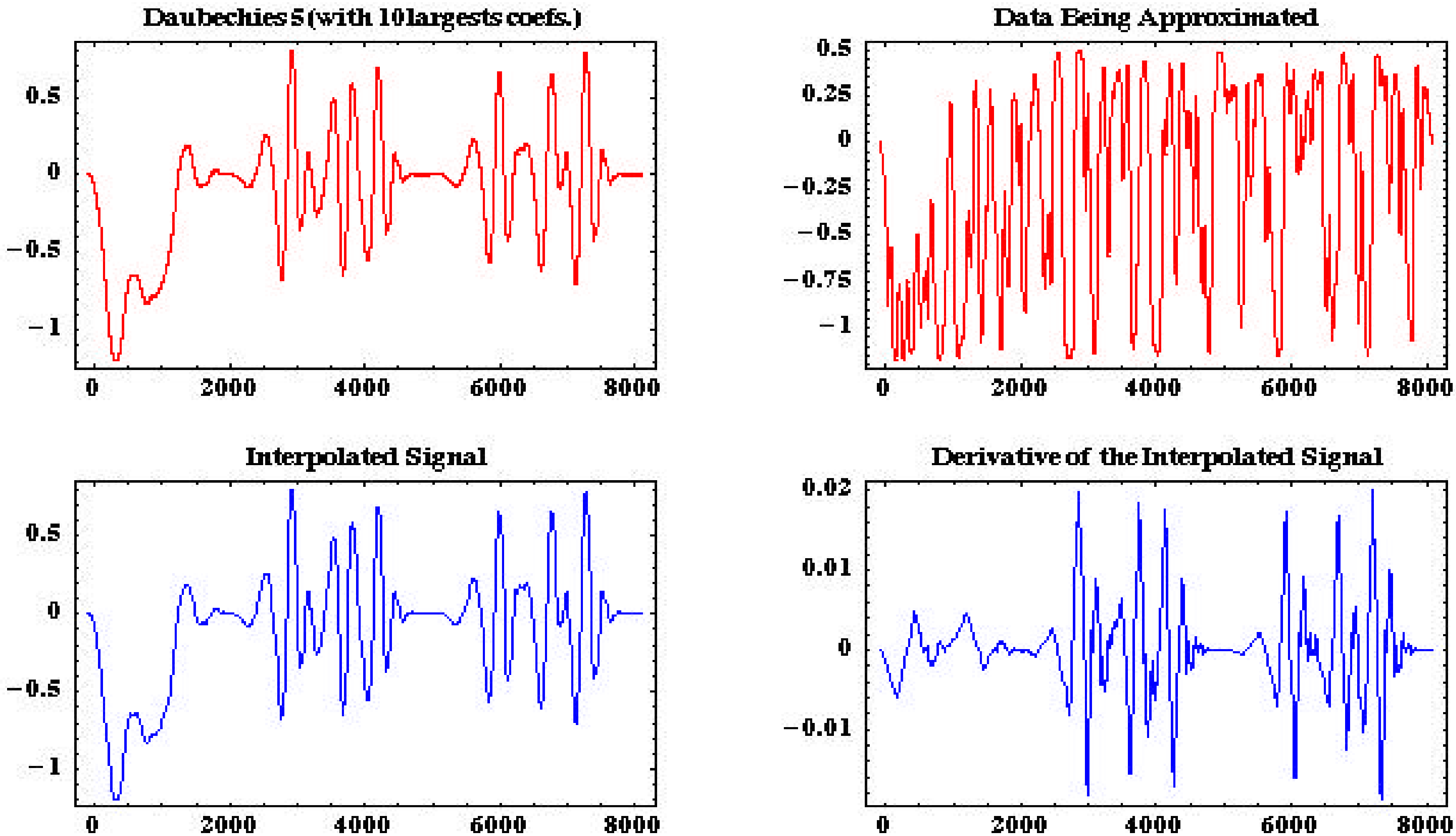
Polymath Research Inc.



Z Pinches

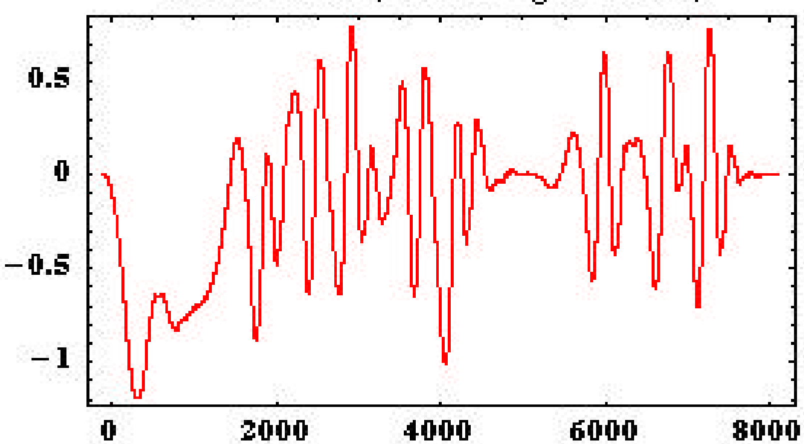
Reconstruction of the LPF RT Weak Mix Data Using the 10 Largest WLT Coefficients

Polymath Research Inc.

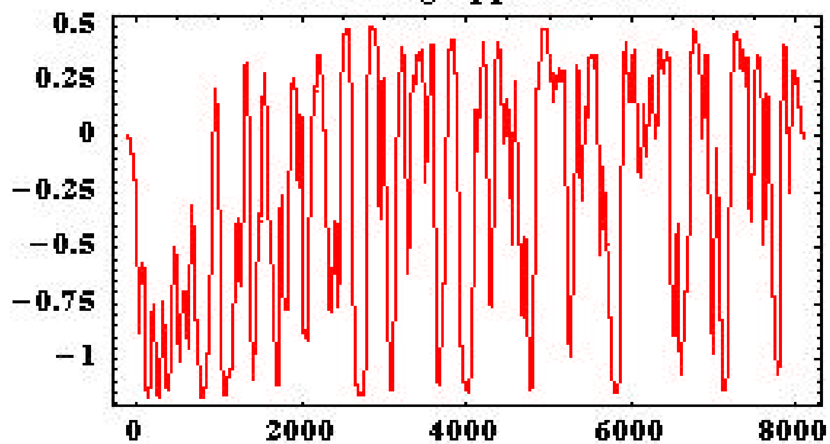


Reconstruction of the LPF RT Weak Mix Data Using the 15 Largest WLT Coefficients

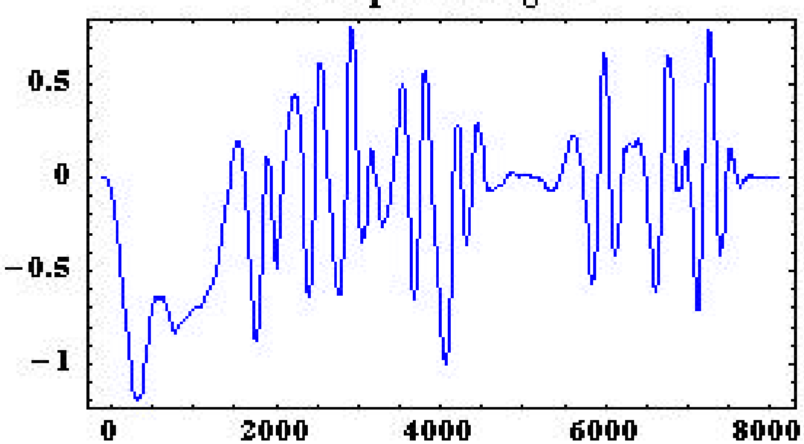
Daubechies 5 (with 15 largests coefs.)



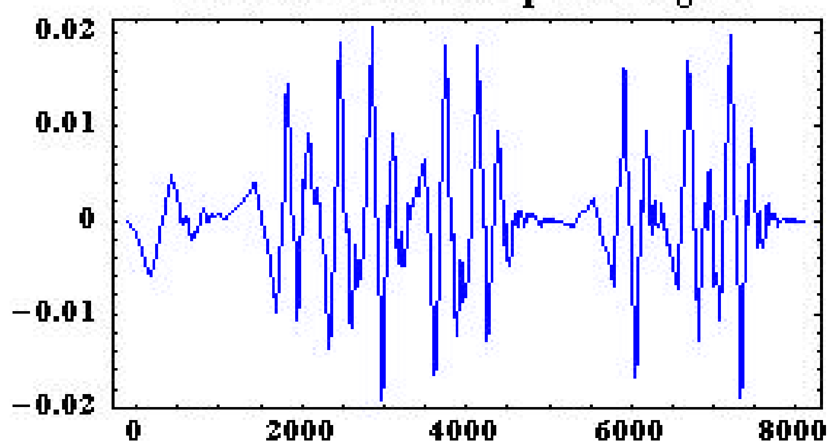
Data Being Approximated



Interpolated Signal



Derivative of the Interpolated Signal



Reconstruction of the LPF RT Weak Mix Data Using the 20 Largest WLT Coefficients

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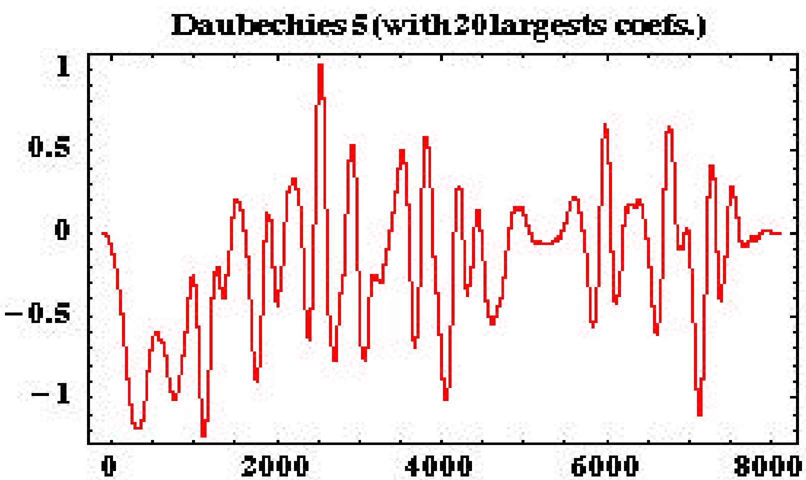
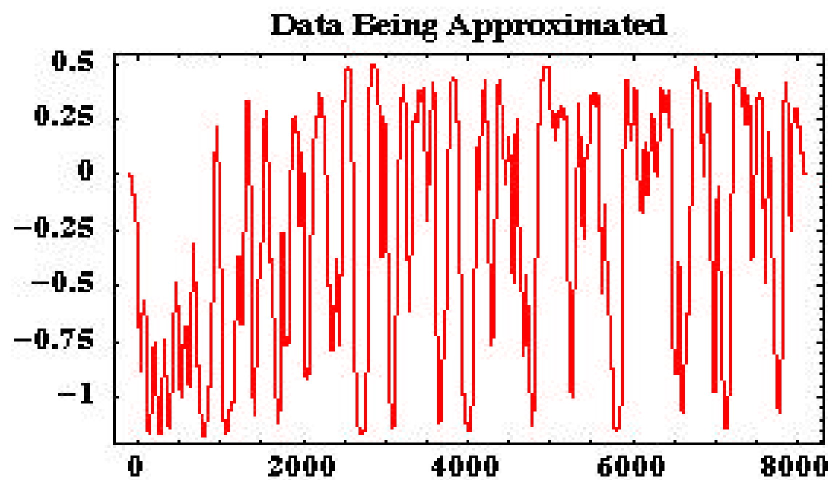
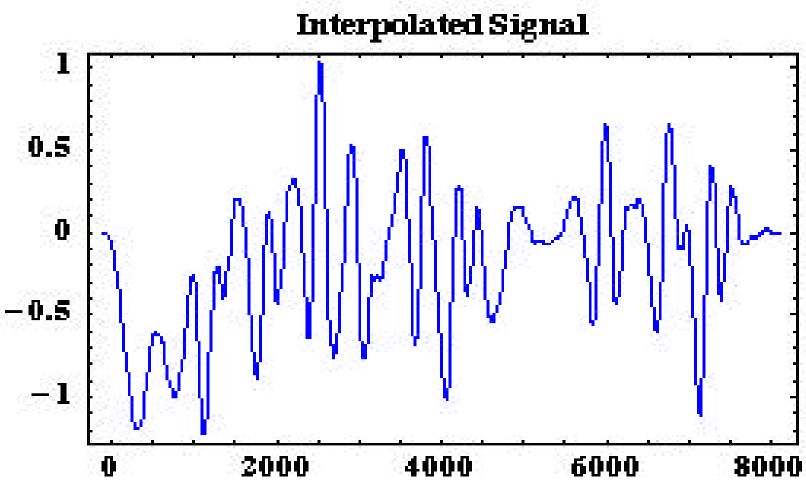
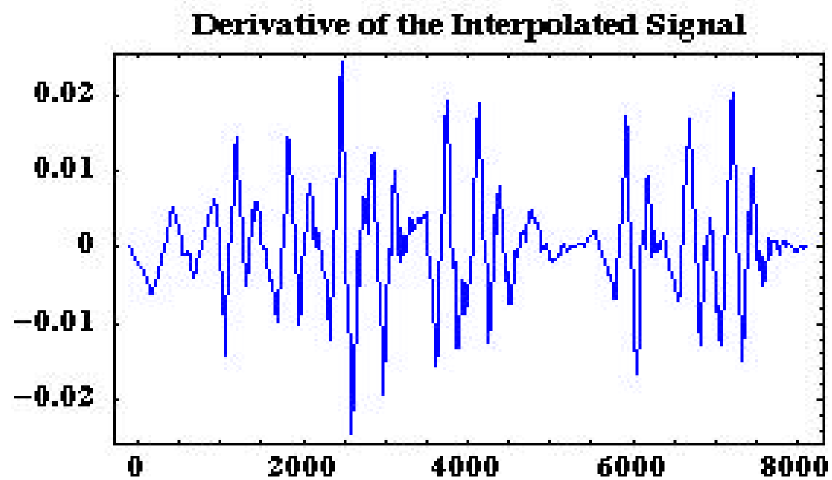


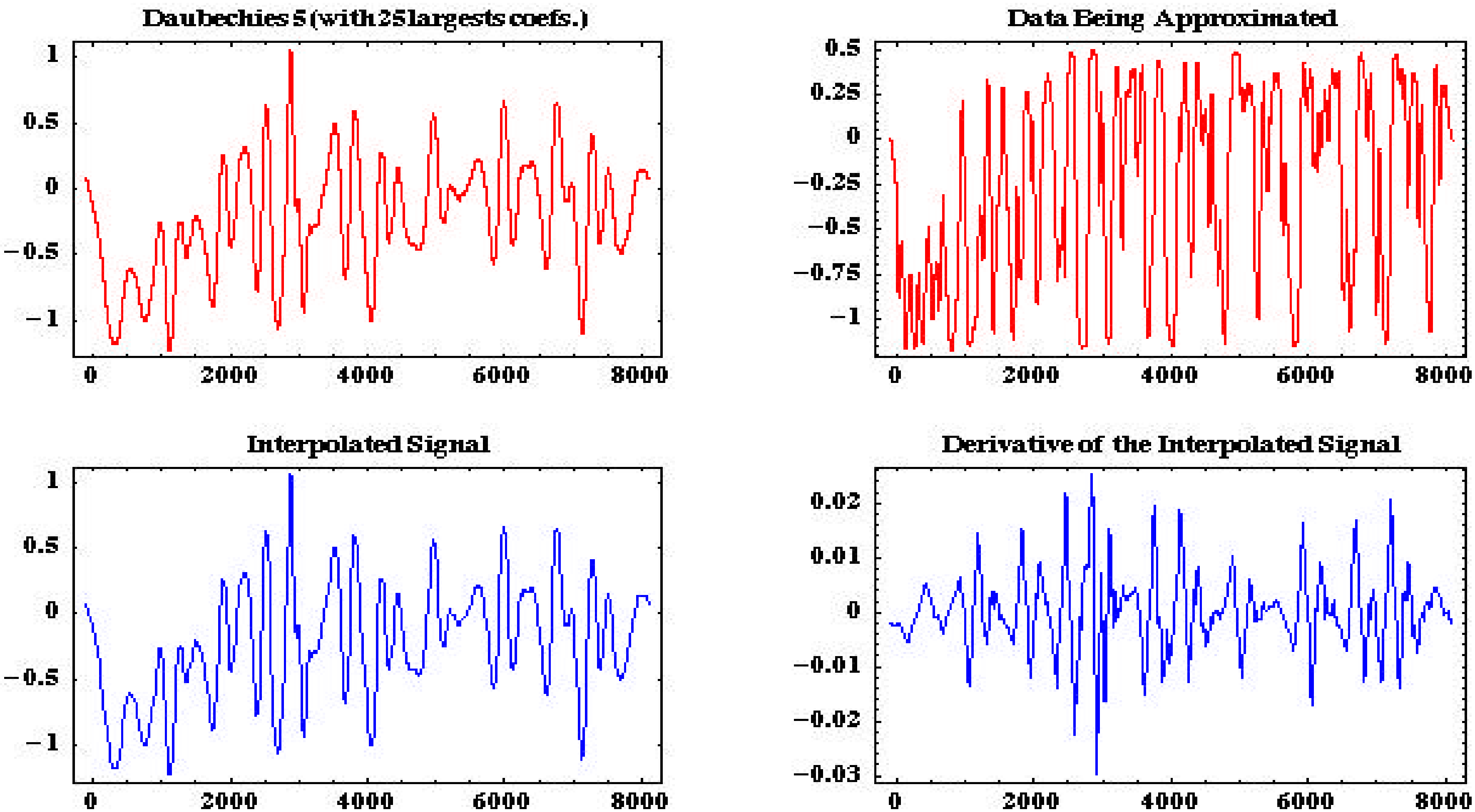






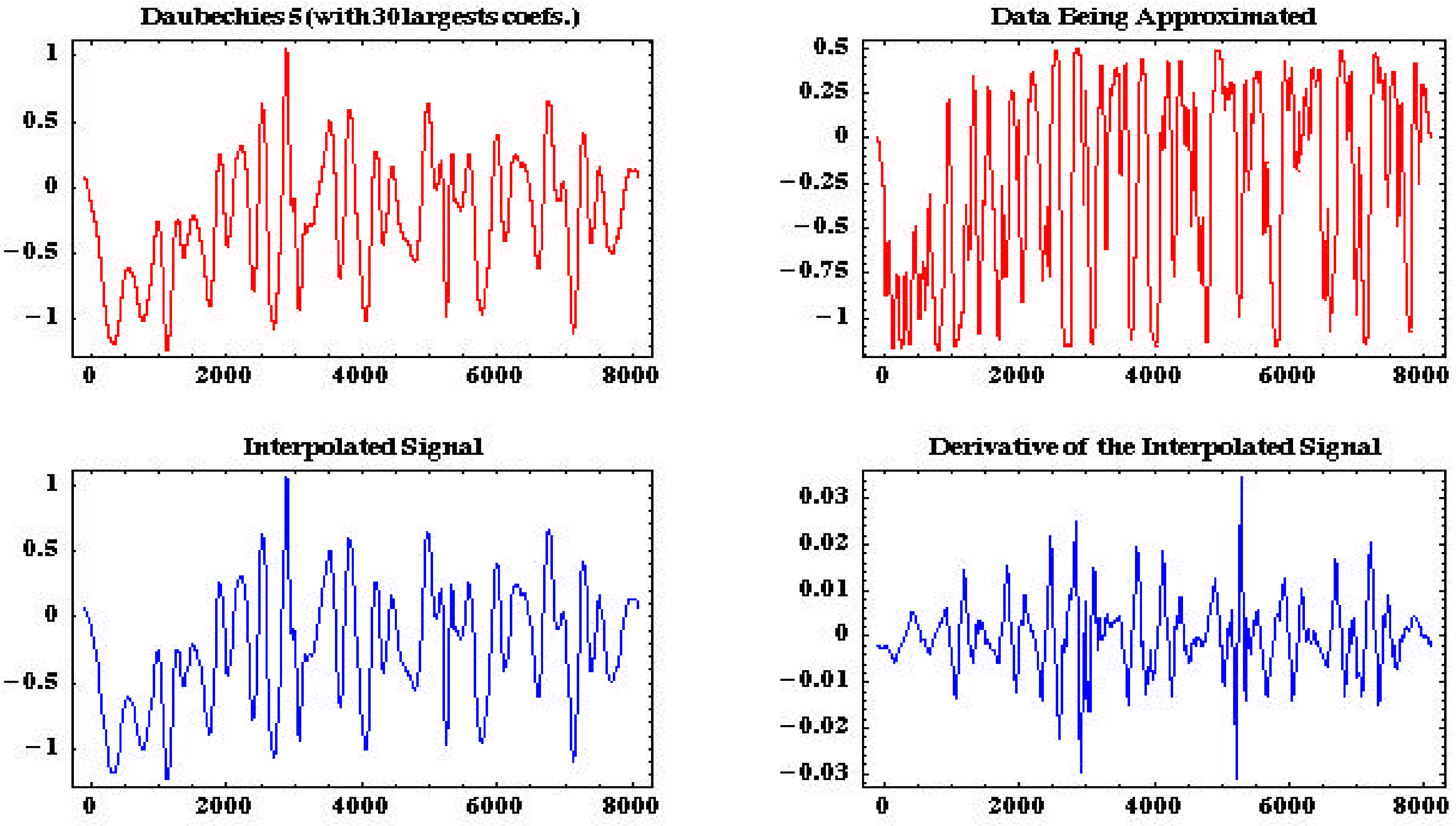
Reconstruction of the LPF RT Weak Mix Data Using the 25 Largest WLT Coefficients

Polymath Research Inc.



Reconstruction of the LPF RT Weak Mix Data Using the 30 Largest WLT Coefficients

Polymath Research Inc.



Haar Wavelet Amplitudes vs Variance on Each Scale of MRD at 35 cm: Highly Correlated Structures

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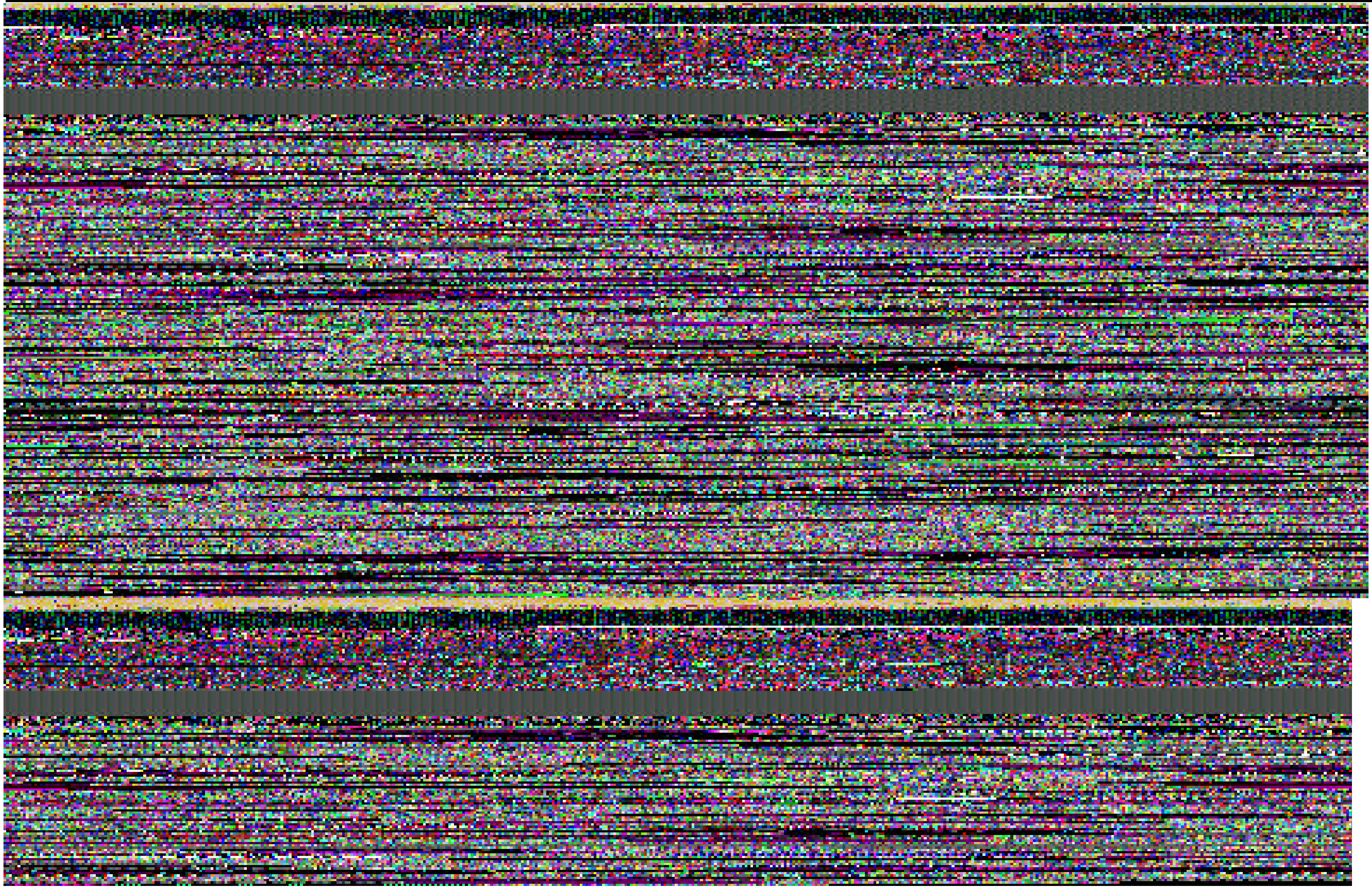
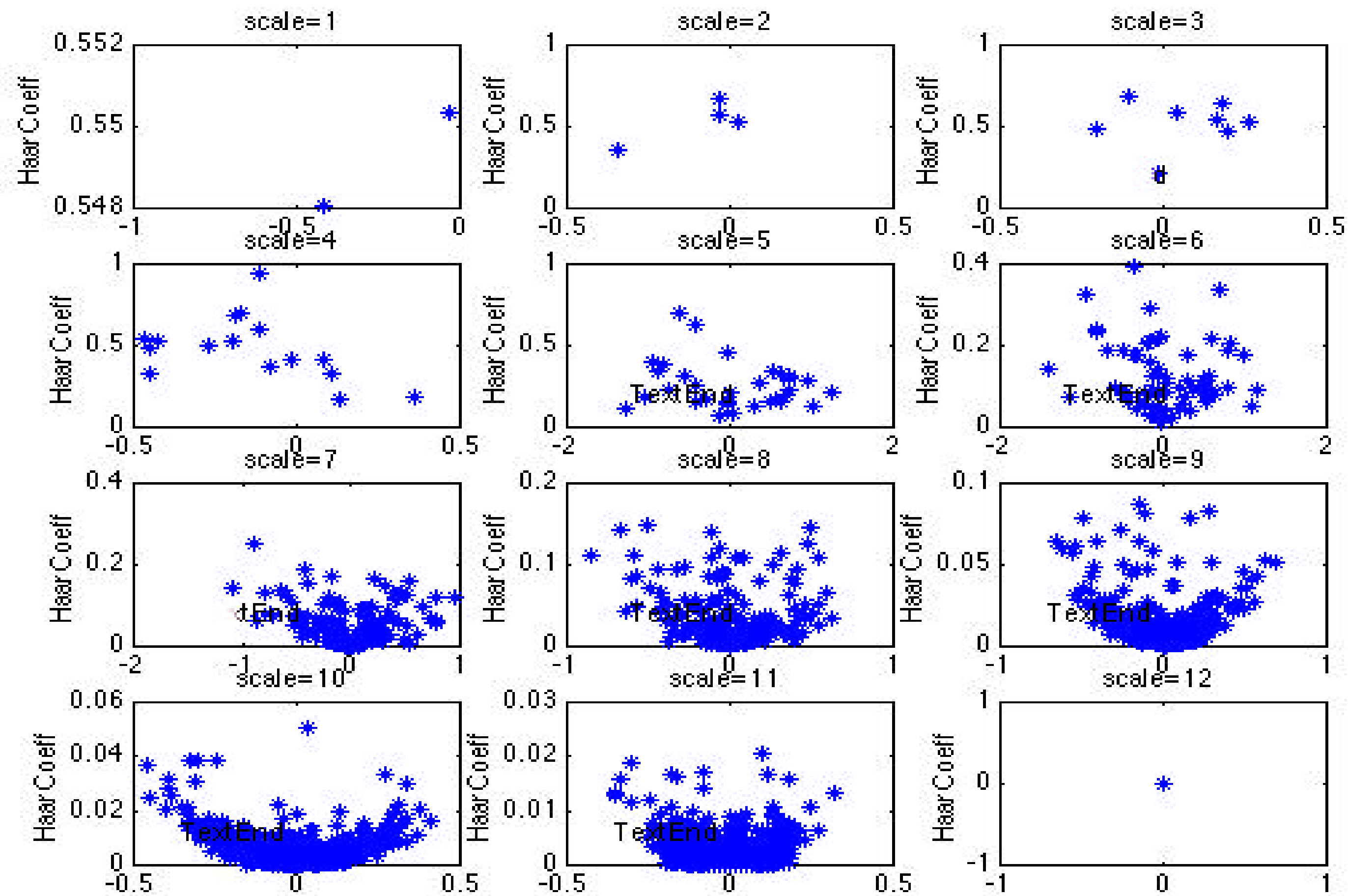


Haar Wavelet Amplitudes vs Variance on Each Scale of MRD at 2 cm: Not So Correlated!

Polymath Research Inc.



Conclusions Based on the LPF RT Weak Mix Data's WLT Analyses

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- **There are considerable differences between the nature of the fluctuations or the "turbulence" at 2 cm and at 35 cm.**
- **Low pass filtering the data to eliminate the electronic detector noise gives rise to clean amplitude separations between scales at the finer levels of MRD.**
- **The correlation between wavelet coefficients or features at intermediate scales of MRD can be seen when turbulence mixing is fully present.**
- **Such correlations are not present at 2 cm, for instance.**
- **Would be interesting to compare data from numerous spacings to each other and not just 2 and 35 cm to see how these scale specific correlations develop.**
- **We are developing more comprehensive criteria by which to characterize turbulence and establish predictive tools for mixing, etc., based on specific properties of wavelet coefficient distributions.**

Wavelet Analysis Applied to 2D Mach II Data from a Nested Shell Implosion

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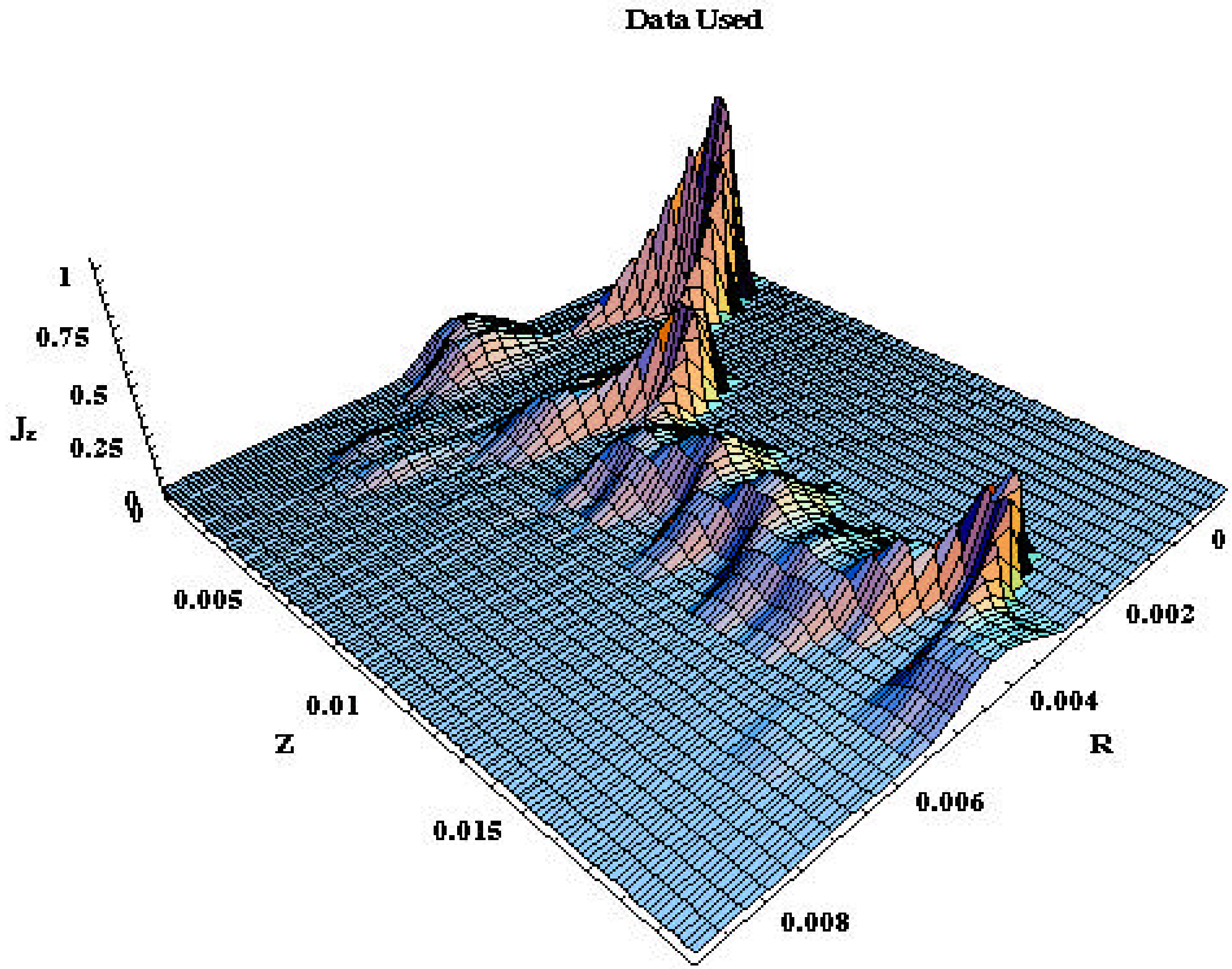

T=210 ns
Z 179
$J_z(R,Z)$
Mach II
Simulation

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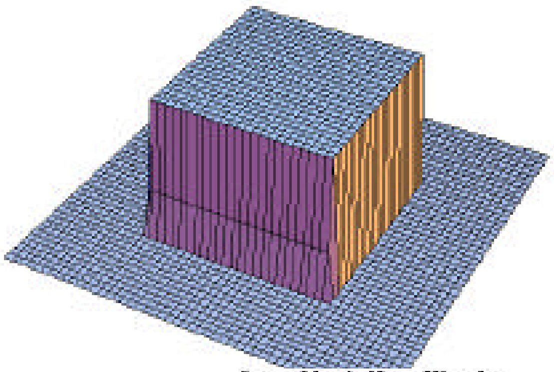
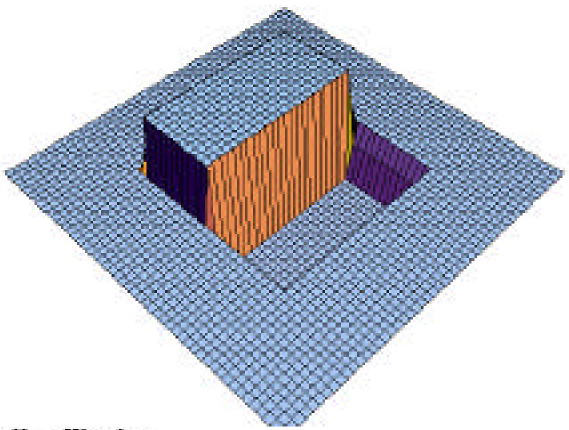
2D Haar Wavelets Look Like This

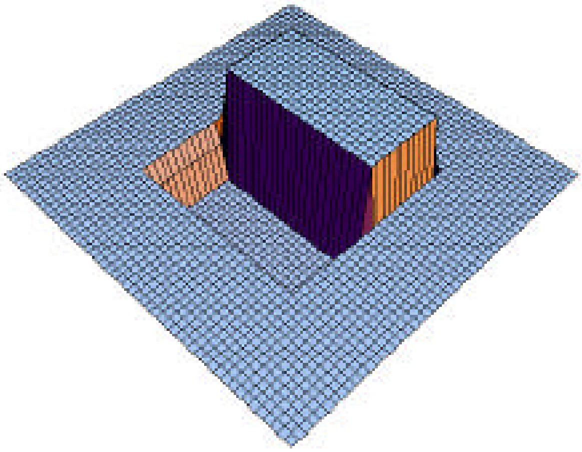
Two-dimensional Haar Scaling Function



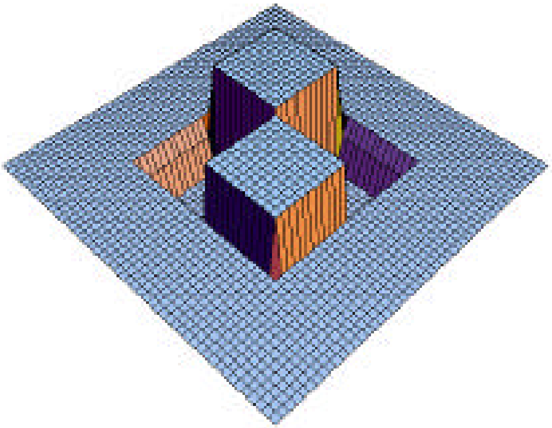
First basic Haar Wavelet



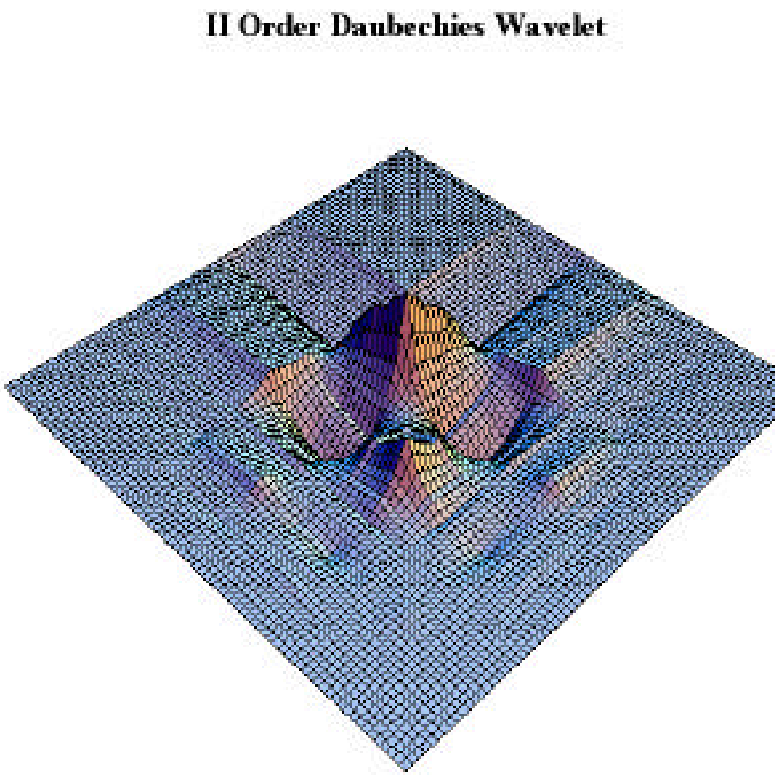
Second basic Haar Wavelet



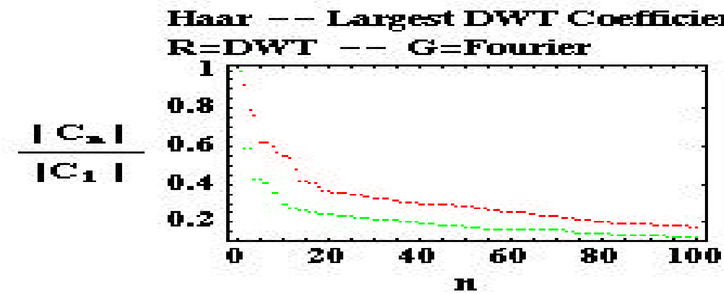
Third basic Haar Wavelet

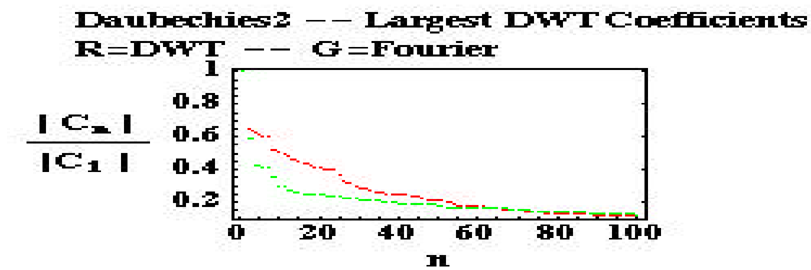


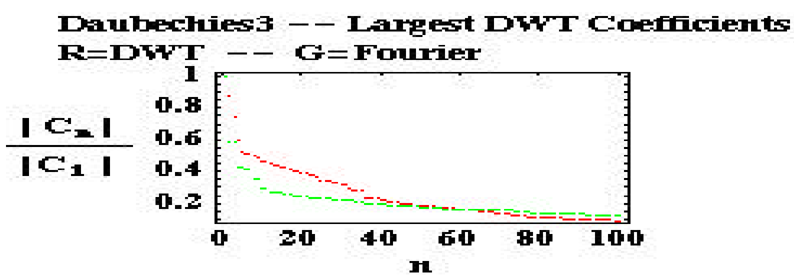
Daubechies Wavelets of 2nd and 3rd Orders Are Fractal!

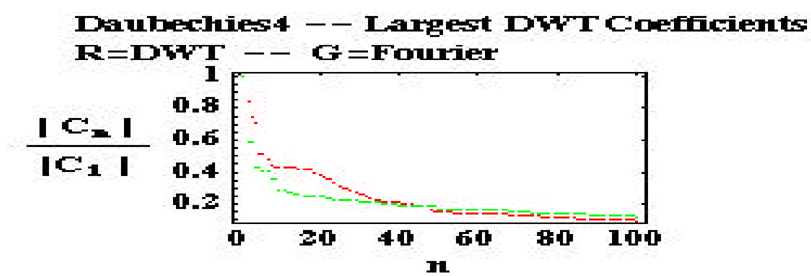


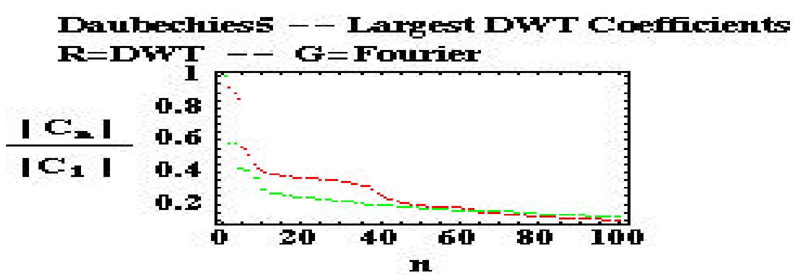
A Comparison of 5 Daubechies with FFTs: The Coefficients



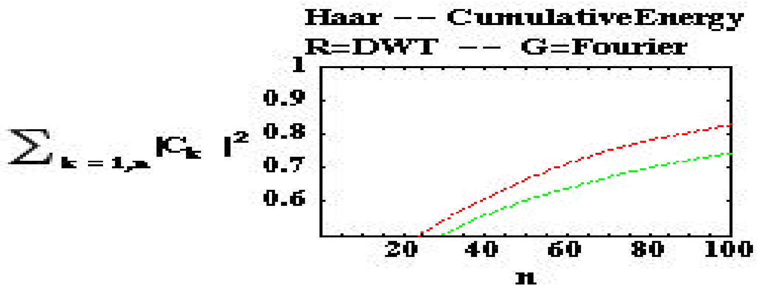


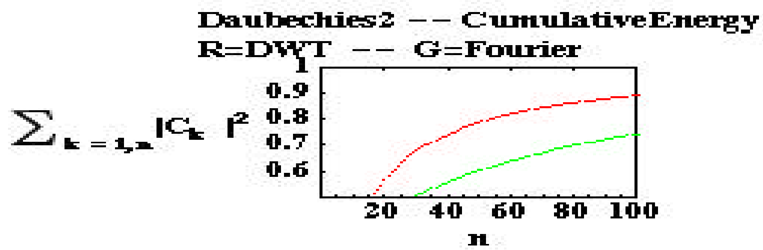


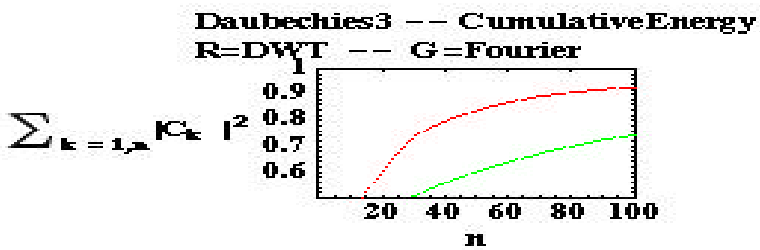


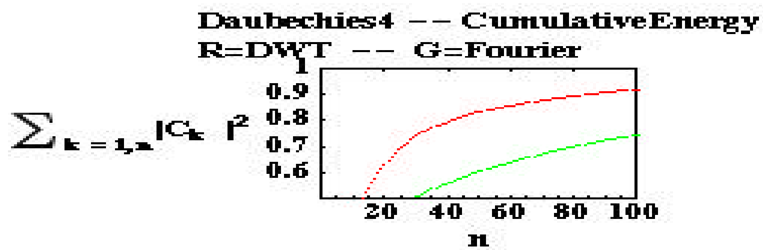


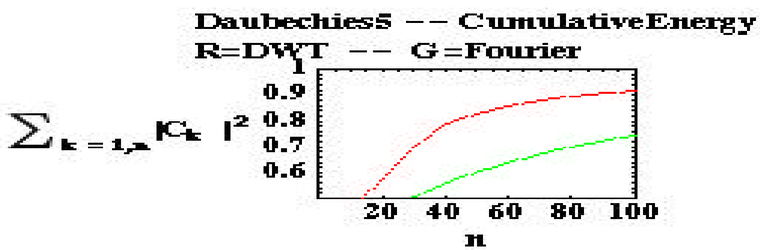
Speed by which the Approximations Converge to the Full Signal: Proper Wavelets Always beat FFTs











MRD Using Daubechies 3 WLTs

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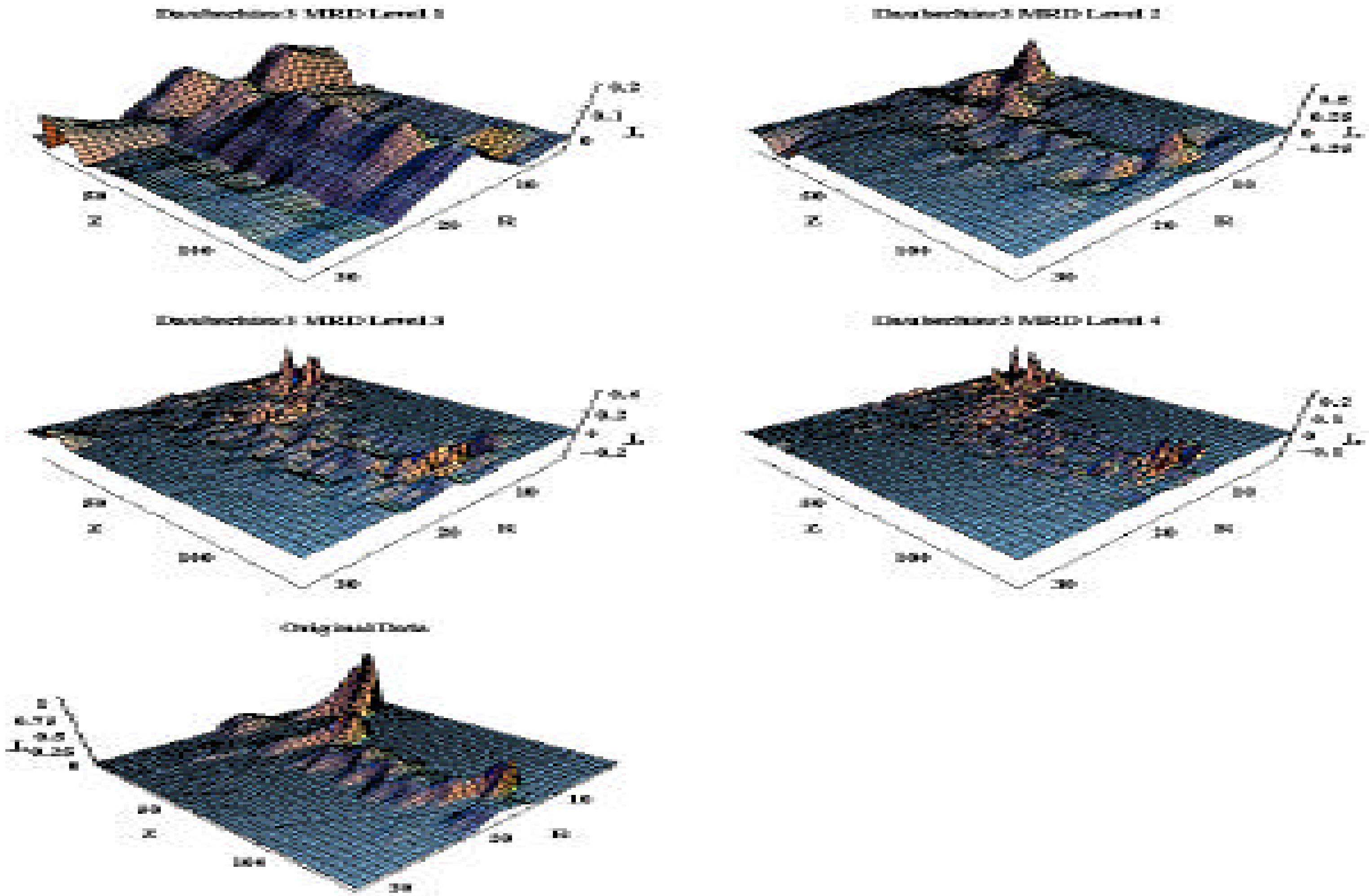

Contour Representations of the Various MRD Levels and the Data

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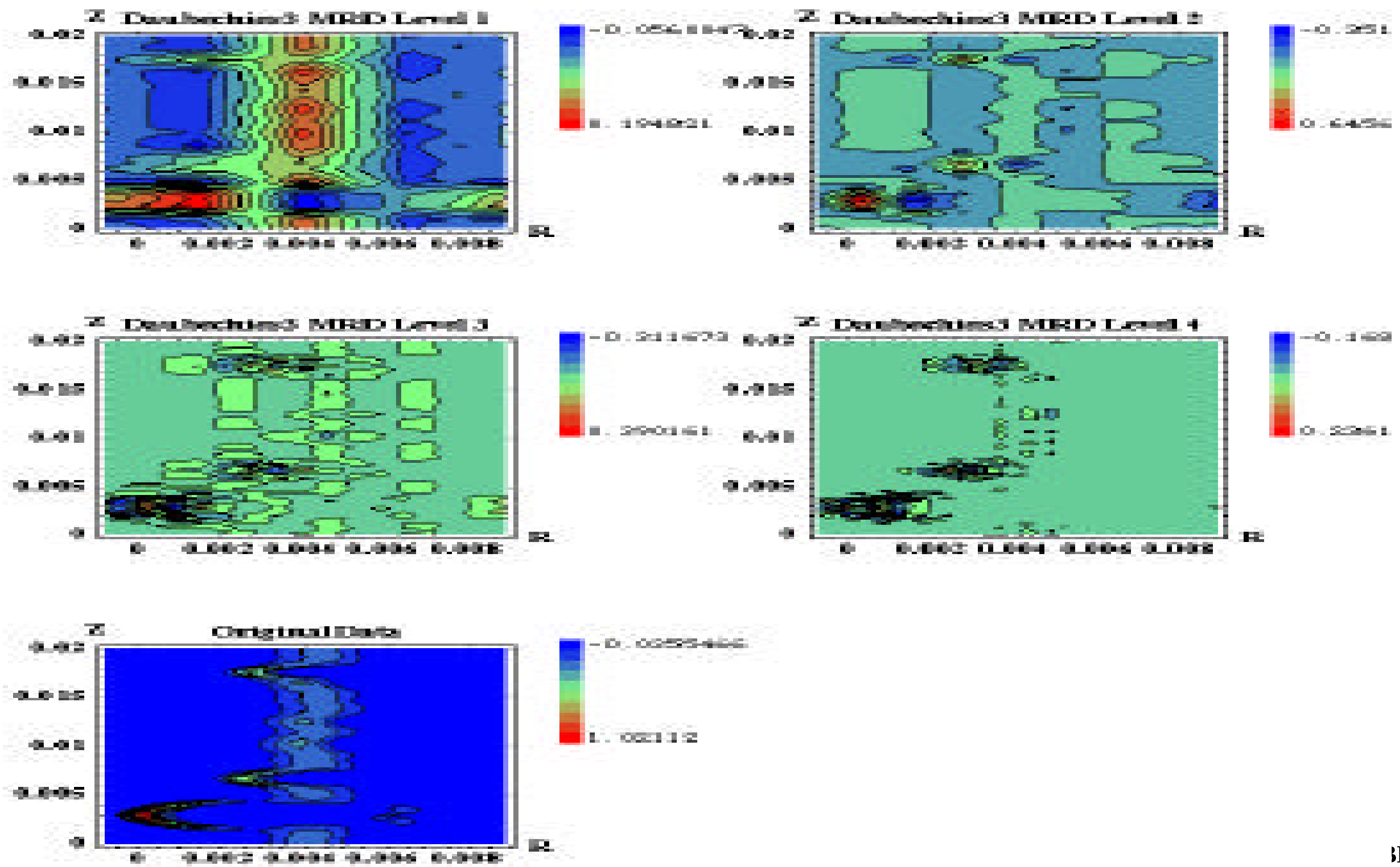

Error Using Just 1 Level in MRD Using Daubechies 3 WLTs

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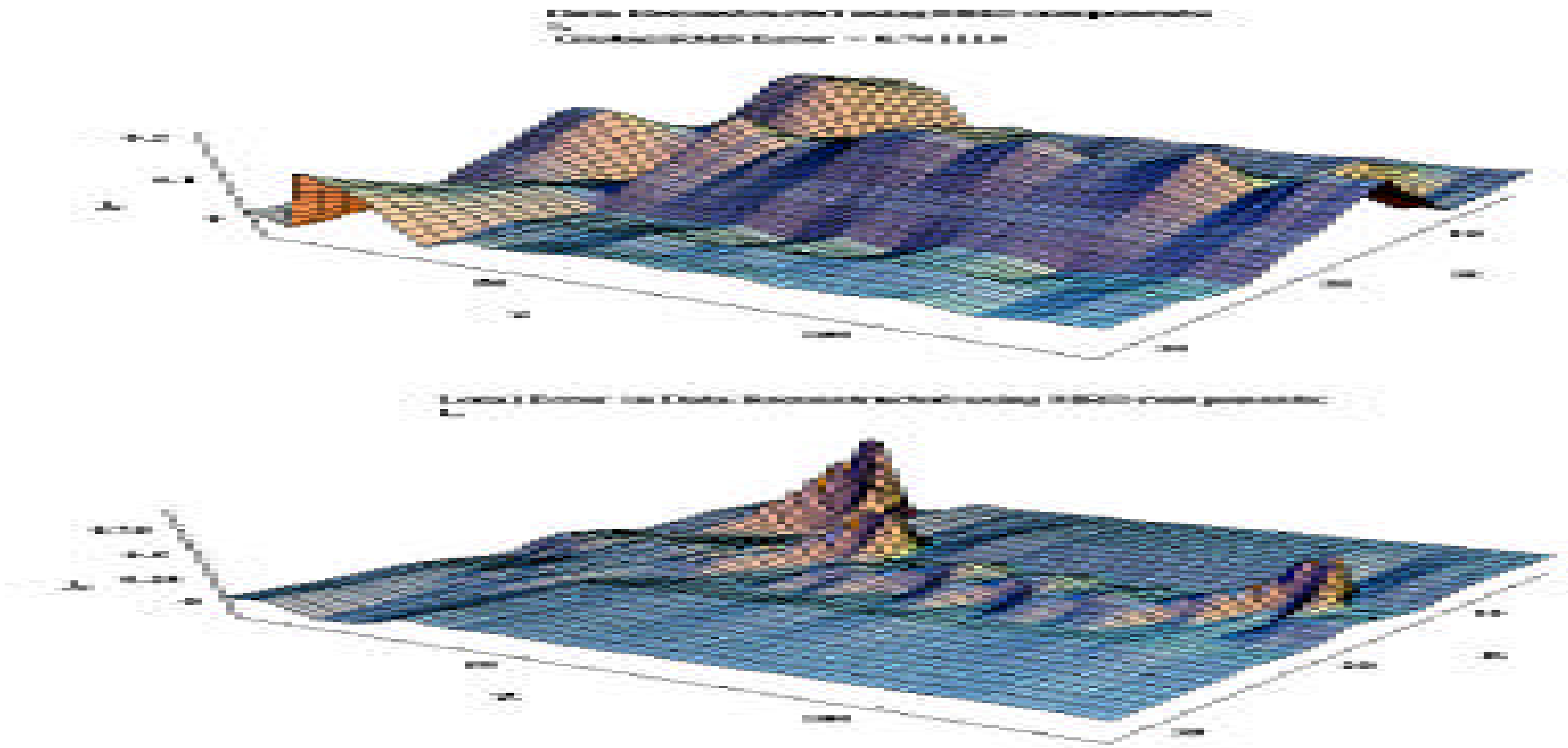
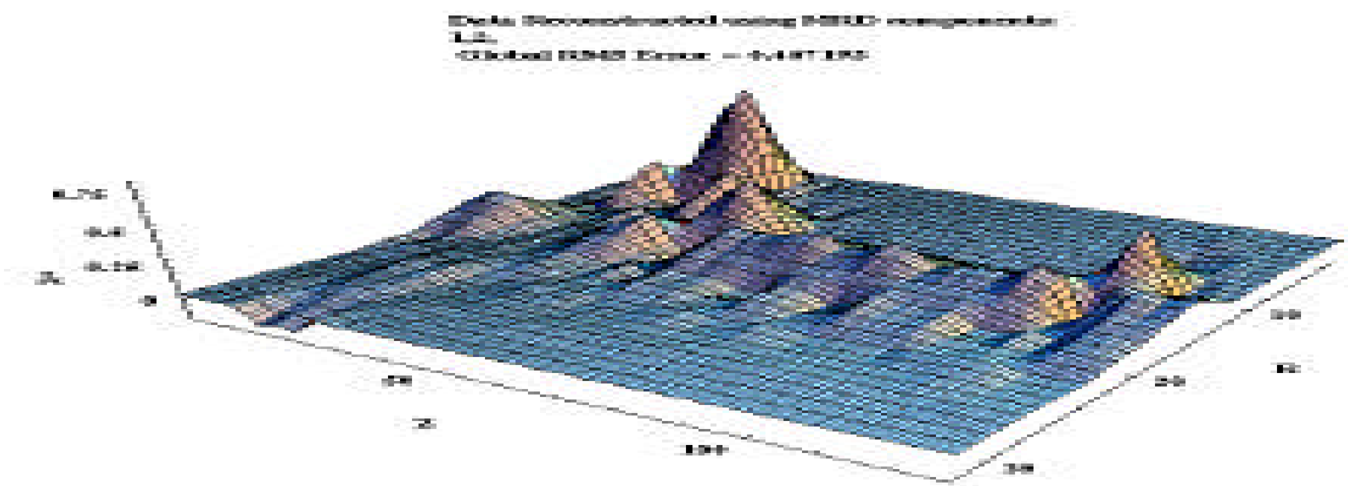
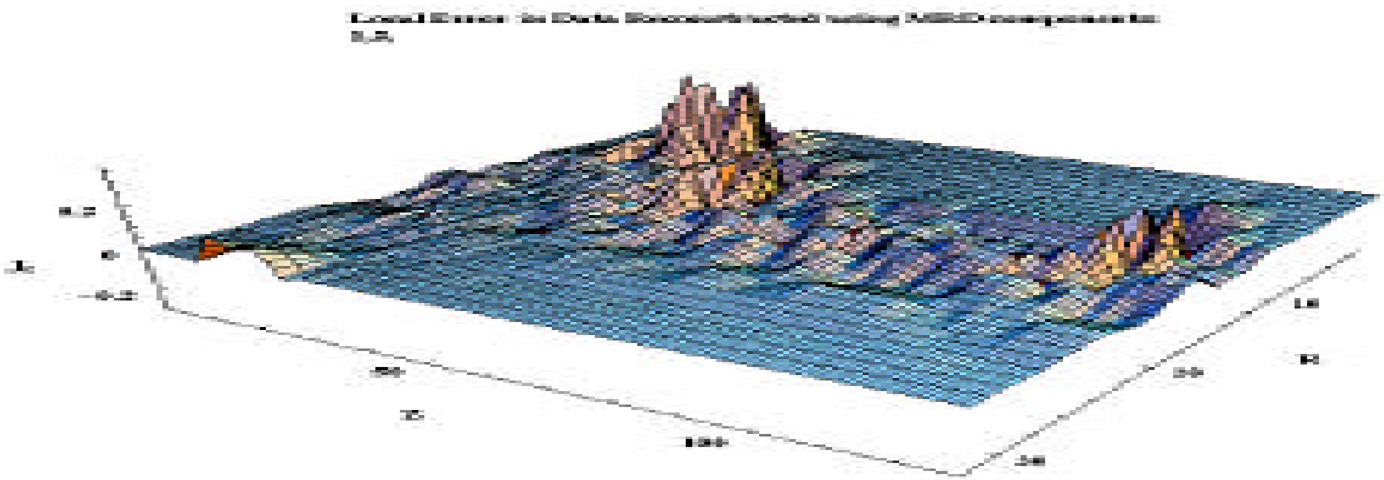


Error Using 2 Level in MRD Using Daubechies 3 WLTs

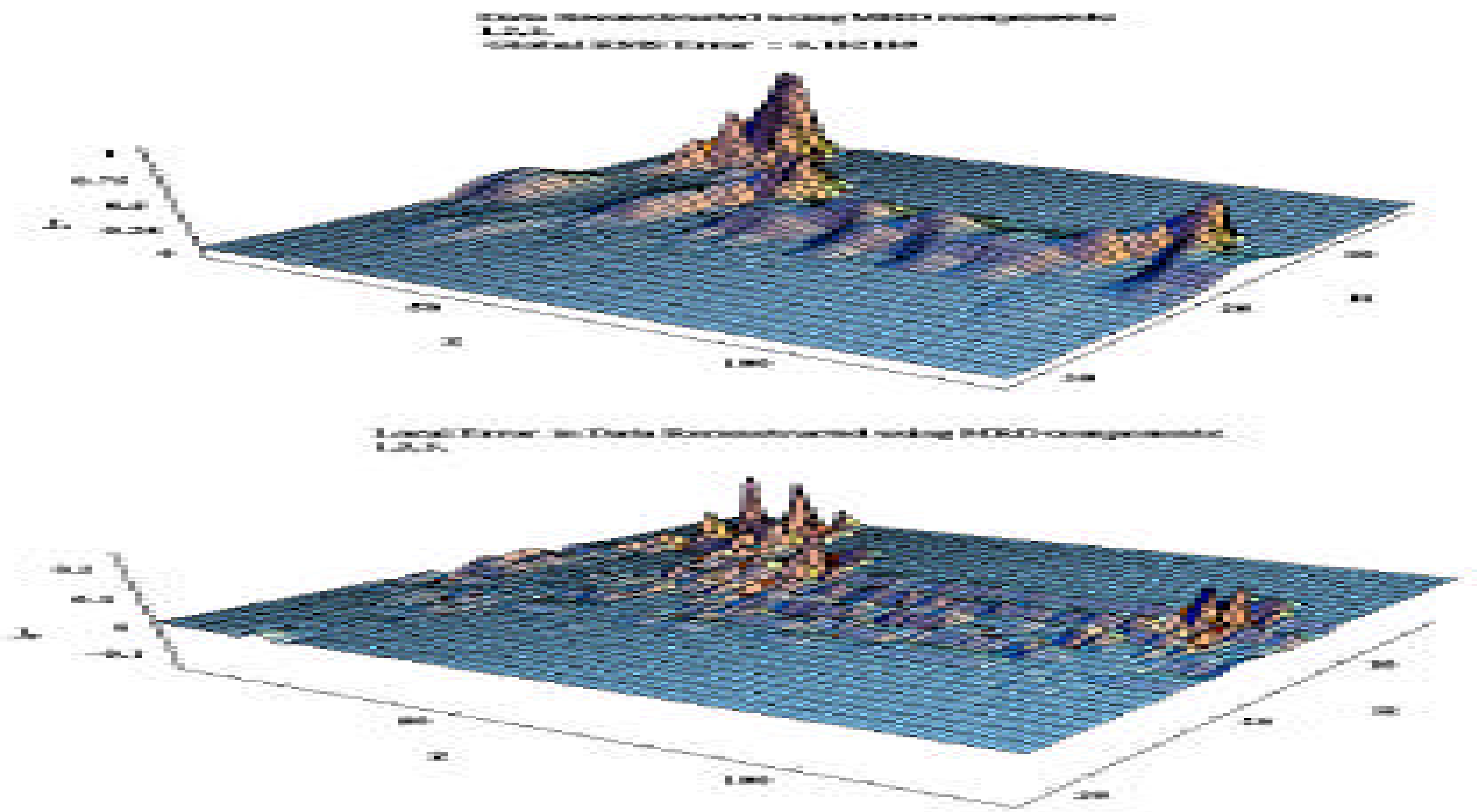
Polymath Research Inc.



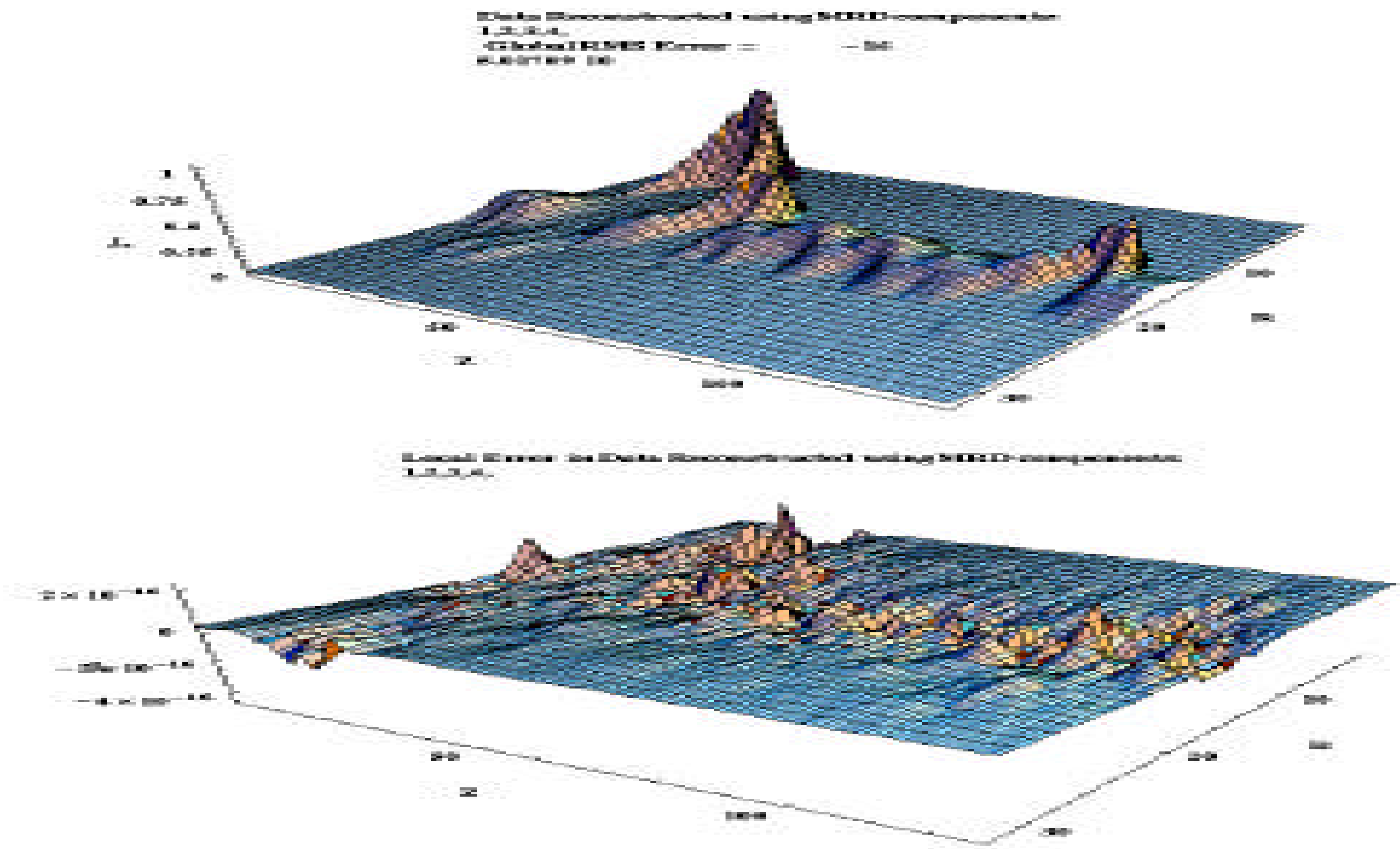


Error Using 3 Level in MRD Using Daubechies 3 WLTs

Polymath Research Inc.



Error Using 4 Level in MRD Using Daubechies 3 WLTs



RMS Error in FFT and the first 5 Daubechies Representations

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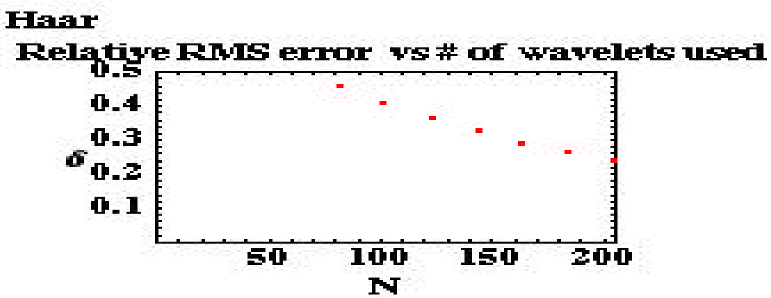

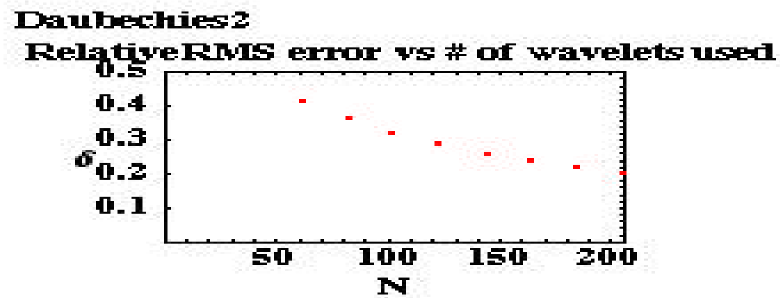

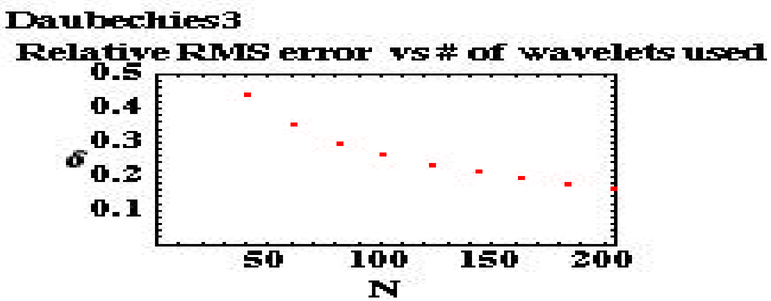

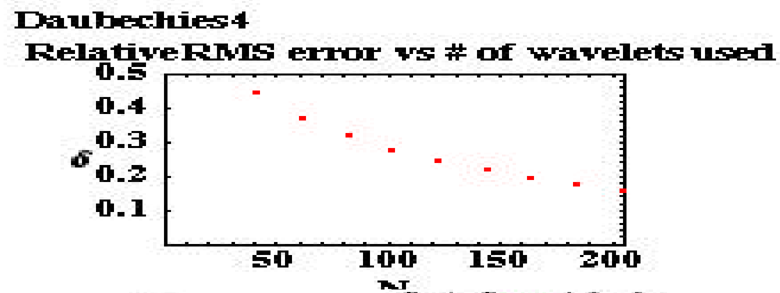

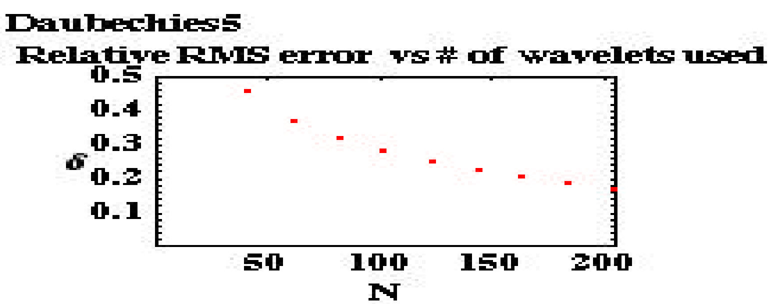

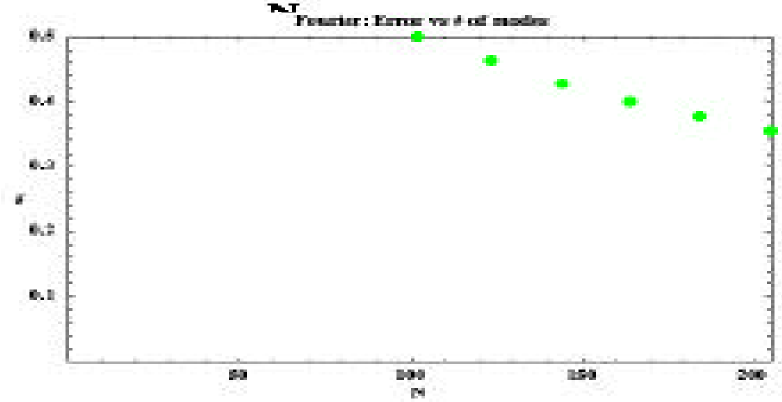

Overall Conclusions

- This is an exciting area of research.
- Wavelet denoising, data compression and fast pattern detection are genuine advances on spectral Fourier transform techniques.
- Wire array Z pinches are an ideal setting for the utilization of these tools since we have structures developing on different scales which we have to understand in order to optimize the implosions.
- Numerical benefits are many as well. Primarily for adaptive rezoning and remeshing applications in fluid and kinetic codes. Much more work remains to be done here.
- We have developed Mathematica notebooks that do all this analysis given an ascii file. These are available to anyone seriously interested in using them.